PHYS 210B: Nonequilibrium Statistical Mechanics
Fall 2022

Problem Set 3

1) Short Answer — Shear Dispersion (Kelvin)

a) Calculate the scaling of the mean-square displacement of a Brownian particle in the limit of drag → 0. Explain the physics of your result. Contrast the finding with "the usual".

b) Now calculate the decorrelation time of a particle in a sheared flow $\underline{v} = v(x)\hat{y}$. Assume the particle is scattered in $\hat{x}$ — i.e. the usual random force is $\underline{f} = f\hat{x}$. The decorrelation time is defined by $\langle \delta y^2(\tau_c) \rangle = l_0^2$, where l_0^2 is an FOM. Note $\tau_c \neq \tau_{ac}$. Discuss the Physics in light of Part a.

2) Consider the Kramers problem in the viscous limit, as discussed in class. Assume some number of particles are placed in well A.

a) Calculate the probability of a particle in well B to return to well A, once B is occupied.

b) Under what conditions will the net reaction flow vanish?

3) Planet formation in an accretion disk is thought to occur by a process of sedimentation in a turbulent flow. Large dust grains sediment out of the disk flow. The disk flow is turbulent.

Assume:

i) The disk $v_\theta(r)$ flow is Keplerian — balance centrifugal force and gravity, due to some control object of mass M.

ii) The central object produces a vertical gravitational field.

iii) The disk is turbulent. Assume there exists a spectrum of ambient vertical velocity fluctuations. You may take these as delta correlated in time, and homogeneous in space (unrealistic, but convenient).

a) Using what you have learned, derive a Schmoluchowski equation for the dust density $n(z)$. Assume the dust is heavy, relative to the gas. z is vertical.

b) Estimate the thickness of the settled dust layer. This is not set by thermal effects.

c) Extra credit: Estimate under what conditions the dust sheet will break up into rings leading to planet formation.

4) (a) Calculate the frequency dependent mobility of an ensemble of Brownian particles.

(b) Now calculate the frequency dependent mobility for a collisional gas, by the methods discussed earlier in the course.

(c) Compare your results for a, b. Discuss correspondences.

5) How would you approach the Kramers problem in the “weak viscosity” limit? Set up the problem and at least make some estimates? How does transition probability scale with drag?

N.B. Resist the urge to copy the well-known solution from a source!

6) Write a brief critique (1-2 pages) of the Kubo formalism, from the perspective of actual calculations.