PHYS 210B: Nonequilibrium Statistical Mechanics
Fall 2022

Problem Set 1

1) a) Prove the Lemma discussed in class, which was used to prove the Boltzman H-Theorem.

   b) Show that the Boltzmann collision operator conserves number, momentum, energy.

2) a) Derive the vorticity evolution equation for a fluid in motion, for which $P \neq P(\rho)$.

   b) What are the implications for conservation of circulation in such a fluid, assuming it to be inviscid?

3) Estimates:

   a) Derive the shear viscosity and thermal diffusivity of a hard sphere gas by heuristic methods.

   b) Consider a heavy particle of mass M radius d_2 in a gas of light particles of mass m, radius d_1 ($d_1 < d_2$, $m \ll M$).

      i) What is the mobility of the heavy particle?

      ii) What is the deflection length for the heavy particle?

      iii) When will heavy particle energy equilibrate with that of a light particle?

4) Calculate the heat conduction coefficient of a dilute, hard sphere gas using the Krook-model collision operator. Use the method of moment hierarchy truncation.

5) (a) The motion of an electron belonging to a molecule in a rarefied gas may, in some cases, be replaced by that of a harmonic oscillator: it is determined by

$$\frac{dx}{dt} = \frac{p}{m}, \qquad \frac{dp}{dt} = -m\omega_0^2 x - eE(t),$$

where x and p denote the radius vector and the momentum of the electron within the molecule respectively, m the mass, $-e$ the electric charge, ω_0 the characteristic angular frequency, and $E(t)$ an external electric field. Show that the average $\bar{f}(x,p,t)$ of the electron distribution function $f(x,p,t)$, taken over all possible values of the time and of the position at which collisions occur, obeys the equation

$$\frac{\partial \bar{f}}{\partial t} + \frac{p}{m} \cdot \frac{\partial \bar{f}}{\partial x} + \{-m\omega_0^2 x - eE(t)\} \cdot \frac{\partial \bar{f}}{\partial p} = -\frac{\bar{f} - f_0}{\tau}$$

by assuming that collisions between molecules occur with a mean free flight time τ, and that the electron distribution function immediately after a collision reduces to a given distribution function $f_0(x,p,t)$.

(b) Derive the low frequency electric current in the limit of small τ. Note $E = E(t)$ here.

(c) What is the effective Ohm's Law for this system? Discuss.

(d) For the cases of large τ (weak collisionality) and $\omega_o^2 \to 0$, propose and implement a mean field approach to computing how f evolves.