Physics 210b

Plasma Response, Vlasov Equation and Landau Damping → a tale of Vlasov, Landau and Sagdeev (next).

→ For detailed treatment, see 218a notes, Fall 2018 (Dept. site).

→ discussion here appropriate to more general study of kinetics.

a.) Recall : Kubo Formalism

Linear Response, via Liouville Eqn
⇒

Transport 'Coefficient' ⟺ Correlation Function

More generally, linear response of distribution function ⟺
Collective response

Simple example of collective response function is:

$\epsilon(\underline{k}, \omega)$ → dielectric function
i.e. response to test external electric field

$\epsilon(\underline{k}, \omega)$ generalizes $\epsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2}$

Why care:

- collective resonance / modes

$$\epsilon(k, \omega) = \epsilon_r(k, \omega) + i\,\epsilon_{Im}(k, \omega)$$

so

$$\langle E^2 \rangle_{k,\omega} = \frac{\langle \rho_{ext}^2 \rangle_{k,\omega}}{|\epsilon(k,\omega)|^2} \left(\frac{4\pi}{k}\right)^2$$

⇒ collective modes (collective resonances)
(electrostatic)

where $\epsilon_r(k, \omega) \to 0$

$\epsilon_{Im}(k, \omega)$ "small"
(i.e. weakly dissipative)

Ultimately, linear response

is basic element in medium/system collective response.

and: $\epsilon(\underline{k},\omega) = 0 \Rightarrow \omega = \omega(\underline{k})$ (waves, modes)

$\epsilon_{Im}(\underline{k},\omega_r) \Rightarrow$ dissipation (growth or damping)

growth $\Rightarrow$ collective instability, of great interest

So:

Linear Response $\rightarrow$ transport (at, near eqbm)

Linear Response $\rightarrow$ collective dynamics (ultimately strongly non-equilibrium $\Rightarrow$ evolution, mode coupling, turbulence).

[Kubo formalism is, at best, 1/2 story.]

One example of collective response function, i.e. $\epsilon(k,\omega)$, role is energy theorem → i.e. Poynting

Can write wave energy theorem, for waves where $\epsilon_r(\underline{k},\omega) \to 0$

$$\partial_t W + \underline{\nabla}\cdot\underline{S} + Q = 0$$

$\partial_t W$ ↓ wave energy density

$\underline{\nabla}\cdot\underline{S}$ ↓ wave energy density flux

Q ↓ Dissipation

obvious analogy with EM Poynting Theorem

$$\partial_t\left(\frac{\underline{E}^2}{8\pi} + \frac{\underline{B}^2}{8\pi}\right) + \underline{\nabla}\cdot\left(\frac{c}{4\pi}\underline{E}\times\underline{B}\right) + \langle\underline{E}\cdot\underline{J}\rangle = 0$$

here W, $\underline{S}$, Q are specified by $\epsilon(\underline{k},\omega)$.

i.e.
$$W = \omega_{\underline{k}} \left.\frac{\partial\epsilon_r}{\partial\omega}\right|_{k,\omega_k} \frac{|E_k|^2}{8\pi}$$

$$\underline{S} = -\omega_k \left.\frac{\partial\epsilon}{\partial\underline{k}}\right|_{k,\omega_k} \left(|E_k|^2/8\pi\right)$$

$$Q = \omega_k \underbrace{\epsilon_{IM}(k, \omega_k)}_{\text{dissipation}} \frac{|E_k|^2}{8\pi}$$

Where from:

a.) $\int d^3x \, \underline{E}^* \cdot \underline{D}$ → energy density of dielectric medium

then: $$\frac{dW}{dt} = \frac{1}{8\pi} \, \mathrm{Re} \underbrace{\left(\underline{E}^* \cdot \frac{d\underline{D}}{dt} \right)}_{\text{energy in medium builds up by response to external field}}$$

write:

$$\underline{E} = \underbrace{\underline{E}_0(t, \underline{x})}_{\text{envelope (slow)}} \underbrace{e^{i(k_0 \cdot x - \omega_0 t)}}_{\text{carrier - fast}}$$

t → build up of local energy

x → spread of local perturbation

then exploiting space-time scale separation ⇒ above.

see Landau, Lifshitz "Continuous Media"

b.) $\underline{\sigma} \Rightarrow \underline{V}\sigma r$

The point: $\left\{ \text{Linear response and collective (linear) response encode a } \underbrace{\text{lot}} \text{ of information} \right.$

b.) The Vlasov Equation

Aside: How describe plasma?

Recall: $\bar{r} < \lambda_D < l_{mfp} < L$

or

$\bar{r} < \lambda_D < L < l_{mfp}$

(collisionless case) *

As for gas, can write Liouville equation for N particles, i.e.

$$(\partial_t + \mathcal{L}) f = 0$$

"c.f. Physical Kinetics"

Here, have $1/n\lambda_D^3 \ll 1$,

analogous to $nd^3 \ll 1$ for BBGKY.

Common element: $\boxed{\text{Diluteness}}$

so, via similar methods, can close and simplify BBGKY Hierarchy for plasma, yielding "Boltzmann Equation" for plasma:

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla} \underline{f} + \frac{q}{m} \left(\underline{E} + \frac{\underline{v} \times \underline{B}}{c} \right) \cdot \underline{\nabla}_v f = C(f)$$

For electrostatic interaction

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla} \underline{f} + \frac{q}{m} - \underline{\nabla} \phi \cdot \frac{\partial f}{\partial \underline{v}} = C(f)$$

where ϕ must be self-consistent, so

$$\nabla^2 \phi = -4\pi \rho = -4\pi n_0 q \int d^3\underline{v} \, f$$

Now, what is $C(f)$?

→ recall scattering is long range, and determined by numerous weak/glancing collisions

$$\sigma \sim \left(\frac{e^2}{T}\right)^2 \ln \Lambda$$

↔ $\nu \sim v_{th}/l_{mfp} \sim v_{th}\, n\sigma$

" "

→ $C(f)$ better thought of as Fokker-Planck operator

$$C(f) = -\frac{\partial}{\partial \underline{v}}\left[\underline{F}\, f - \frac{\partial}{\partial \underline{v}} \cdot \underline{\underline{D}}\, f\right]$$

cf. [Landau; Rosenbluth et. al.; Balescu - Lenard]

→ Now, relatively easy to

find parameter regimes where

$\langle f \rangle \simeq \langle f \rangle_{Maxwellian}$

Dynamics $\omega \gg \nu$

i.e. "collisionless dynamics".

In this case, describe by:

Boltzmann Equation, with $C(f) \to 0$

$\Rightarrow$

Vlasov Equation

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla} f - \frac{q}{m} \underline{\nabla} \phi \cdot \frac{\partial f}{\partial \underline{v}} = 0$$

(collisionless Boltzmann)

and system:

$$\boxed{\begin{aligned} &\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla} f - \frac{q}{m} \underline{\nabla} \phi \cdot \frac{\partial f}{\partial \underline{v}} = 0 \\ &\nabla^2 \phi = -4\pi \int d^3v \, f \end{aligned}}$$

$\rightarrow$ Vlasov-Poisson system.

Re: Vlasov Equation / Vlasov – Poisson

- relevant to electrostatics, gravity
  ⇒ cosmology, galaxies

- V. E. is continuity equation for phase space fluid

i.e. $$\frac{df}{dt} = \partial_t f + \underline{v}\cdot\underline{\nabla} f + \frac{q}{m}\underline{\nabla}\phi\cdot\partial_{\underline{v}} f = 0$$

$$\frac{\partial f}{\partial t} + \underline{V}_\Gamma\cdot\underline{\nabla}_\Gamma f = -f\,\underline{\nabla}_\Gamma\cdot\underline{V}_\Gamma = 0$$

(2D fluid, minimally)

- Boltzmann → Vlasov is <u>singular</u> <u>perturbation</u>

i.e. $$\frac{df}{dt} = -\partial_{\underline{v}}\cdot\left[\underline{E}f - D_v\underline{\nabla} f\right]$$

$$\Downarrow$$

$$\frac{df}{dt} = 0$$

much like Navier-Stokes ⇒ Euler.

so caveat emptor!

- Vlasov Equation is yet one more equation in journey thru kinetics.

- Centerpiece problem of Vlasov-Poisson system is Landau Problem → 1D plasma wave

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{q}{m}\partial_x \phi \, \partial_v f = 0$$

$$f = \langle f \rangle_{max} + \delta f$$

$$\partial_x^2 \phi = -4\pi n_0 q \int \delta f \, dv$$

i.e. $\omega \cong \omega_{p0} + ?$ → modes

$\boxed{\text{Landau Problem}}$ → linear and collective response.

→ Landau Problem – why care?

– collisionless damping, due wave-particle resonance

– opens door to kinetic instabilities

– 2 component picture $\begin{cases} \text{waves,} \\ \text{non-resonant particles} \\ \text{resonant particles} \end{cases}$

– Vlasov Equation is nonlinear:

i.e. $E \frac{\partial f}{\partial v}$, where $\phi \sim \int f \, dv$

with

– Vlasov Eqn. conserves entropy

i.e. $S = -\int dv \, f \ln f$

$$\frac{df}{dt} = 0$$

so Damping ?!

– relevant to quasi-particle dynamics.

Collisionless Plasma Waves and Landau Damping I

→ Collective Response / Waves in Vlasov Plasma

– $\omega, kv \gg \nu$

$$f = \langle f \rangle + \delta f$$

$\langle f \rangle$: Collisions, long time ~ Maxwellian

δf: treat as collisionless – Vlasov

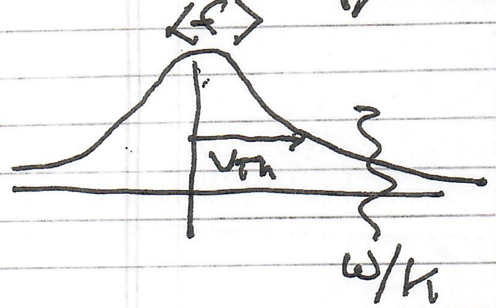
– What of warm plasma waves?

Approach:
- direct calculation
- physics – what Landau Damping means?

- rigorous calculation (II)
- More on physical interpretation

– Direct Calculation (1D)

$$\langle f \rangle = \left(1/\sqrt{2\pi}\, v_{Th} \right) \exp\left(-v^2/2v_{Th}^2 \right)$$



linearizing Vlasov - Poisson:

$$\frac{\partial \tilde{f}}{\partial t} + v\,\frac{\partial \tilde{f}}{\partial x} = -\frac{q}{m}\,\tilde{E}\,\frac{\partial \langle f\rangle}{\partial v}$$

$$\nabla^2 \phi = -4\pi n_0 q \int \tilde{f}\,dv$$

$$f = \sum_{k,\omega} f_{k,\omega}\, e^{i(kx-\omega t)}$$

so

$$-i(\omega-kv)\,\tilde{f}_{k,\omega} = \frac{q}{m}\, i k\, \tilde{\phi}_{k,\omega}\, \frac{\partial \langle f\rangle}{\partial v}$$

$$k^2 \tilde{\phi}_{k,\omega} = 4\pi n_0 q \int \tilde{f}_{k,\omega}\, dv$$

$\Rightarrow$

$$\tilde{f}_{k,\omega} = -k\,\frac{q}{m}\,\frac{\tilde{\phi}_{k,\omega}\;\partial\langle f\rangle/\partial v}{(\omega-kv)}$$

$\hookrightarrow$ $v = \omega/k$?!

$\Rightarrow$

$$k^2 \tilde{\phi}_{k,\omega} = -\omega_p^2 k \int dv\, \frac{\partial\langle f\rangle/\partial v}{(\omega-kv)}\,\tilde{\phi}_{k,\omega}$$

thus

$$\boxed{\epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k}\int dv\,\frac{\partial\langle f\rangle/\partial v}{\omega-kv} \longrightarrow\ !}$$

<u>Dielectric Function for Collisionless Plasma</u>

→ What of Pole at $\omega = kv$?

- recall Vlasov eqn. derived for $\nu \to 0$

$$1/\omega - kv = \lim_{\epsilon \to 0} 1/\omega - kv + i\epsilon$$

(?)

- better, $\boxed{\text{Causality}}$ requires:

$$\phi \sim e^{-i\omega t} \Rightarrow \phi \sim e^{-i(\omega + i\epsilon)t}$$

i.e. $\phi \underset{t \to -\infty}{\to} 0$

so

$$\frac{1}{\omega - kv} = \lim_{\epsilon \to 0} 1/\omega - kv + i\epsilon$$

$$= \frac{P}{\omega - kv} - i\pi\delta(\omega - kv)$$

(Plemelj Formula)

clearly:

$P \Rightarrow$ will recover hydrodynamic response

$-i\pi\delta(\omega - kv) \to \epsilon_{EM}$ "→ Wave Energy Dissipation }" "Landau Damping"

N.B.:

$$Q_k = \frac{|E_k|^2}{8\pi} \omega \, \mathrm{Im}\,\epsilon \Big|_{\omega_k} \rightarrow \text{damping of wave energy}$$

* – of course, for $\partial f_0/\partial v|_{res} > 0 \Rightarrow$ can be growth

– ~~damping~~ growth $\Leftrightarrow \epsilon < 0 \Rightarrow$ analytic continuation (coming)

= wave energy damps; $\rightarrow$ macroscopic

Where? ! $\Rightarrow$ resonant particles

i.e. particles with $v \sim \omega/k$ heating δf?

– How reconcile with $dS/dt = 0$ for Vlasov Eqn.

Proceed with analysis:

$$\epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k}\int dv \frac{\partial \langle f \rangle / \partial v}{\omega - kv}$$

$$= \underbrace{1 + \omega_p^2 \int dv \frac{P}{\omega - kv} \frac{\partial \langle f \rangle}{\partial v}}_{\epsilon_r(k,\omega)}$$

$$\underbrace{- \partial \pi \frac{\omega_p^2}{k|k|} \frac{\partial \langle f \rangle}{\partial v}\Big|_{\omega/k}}_{\epsilon_{Im}(k,\omega)}$$

$$\delta(\omega - kv) = \frac{1}{|k|}\delta(v - \omega/k)$$

Cranking out ϵ_r.

Now, to deal with P:

$$\frac{\partial \langle f \rangle}{\partial v} = \frac{-v}{v_{th}^2} \langle f \rangle$$

$\omega > k v_{th}$ (hydro limit)

$$\frac{P}{\omega - kv} = \frac{1}{\omega}\left(1 + \frac{kv}{\omega} + \left(\frac{kv}{\omega}\right)^2 + \left(\frac{kv}{\omega}\right)^3 + \cdots\right)$$

so

$$\epsilon_r(k,\omega) = 1 - \frac{\omega_p^2}{k v_{th}^2}\int dv \frac{\langle f \rangle}{\omega} v \left(\overset{\times}{1} + \overset{\checkmark}{\frac{kv}{\omega}} + \underset{\times}{\left(\frac{kv}{\omega}\right)^2} + \underset{\checkmark}{\left(\frac{kv}{\omega}\right)^3} + \cdots\right)$$

$$= 1 - \frac{\omega_p^2}{\omega^2} - \frac{3\omega_p^2 v_{th}^2 k^2}{\omega^4}$$

i.e.

$$\langle x^4 \rangle = \int dx \; x^4 e^{-x^2/2}$$

$$= 4 \frac{\partial^2}{\partial \alpha^2}\Big|_{\alpha=1} \int dx \, e^{-\alpha x^2/2}$$

$$= 4 \frac{\partial^2}{\partial \alpha^2}\Big|_{\alpha=1} \left(1/\sqrt{\alpha}\right)$$

$= 3$ (π via normalization)

→ "3" appears from moments of Maxwellian → eqbrm distribution

→ Moments replace/underly eqn. of state

so

$$\epsilon_r(k,\omega) = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + 3\frac{k^2 v_{th}^2}{\omega^2}\right)$$

$\epsilon = \epsilon_r + i\,\epsilon_{Im}$

→ $\epsilon_r = 0$ ⇒ Collective Resonance / Wave

Now:

- should connect to warm plasma wave
- as ϵ derived via $(kv/\omega) \ll 1$ expansion, need determine $\omega(k)$ iteratively.

L.O.:

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + \frac{3k^2 v_{th}^2}{\omega^2}\right)$$

l.o. $\quad \epsilon_r \simeq 1 - \frac{\omega_p^2}{\omega^2}$

$$\omega^{(0)} = \omega_p$$

so

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + \frac{3k^2 v_{th}^2}{\omega_p^2}\right)$$

$$\omega^2 = \omega_p^2\left(1 + 3k^2\lambda_D^2\right) \rightarrow$$

$\Downarrow$

$\gamma \rightarrow 3$, here

[structure agrees with fluid model]

N.B.:

- distribution function $\Leftrightarrow$ E.O.S.!
- dispersion relation identical to warm fluid model $\Leftrightarrow$ $k v_{th} < \omega$ expansion

$\rightarrow \epsilon_{IM}$

$$\epsilon_{IM} = -\frac{\pi \omega_p^2}{k|k|} \frac{\partial \langle f \rangle}{\partial v}$$

so dissipated wave energy:

$$Q_k = \omega \epsilon_{IM} \frac{|E_k|^2}{8\pi}\Big|_{\omega_k}$$

$$\boxed{Q = -\omega_k \frac{\pi \omega_p^2}{k|k|} \frac{\partial \langle f \rangle}{\partial v}\Big|_{\omega_k/k} |E_k|^2/8\pi}$$

and

$$\frac{\partial W_k}{\partial t} + \underline{\nabla} \cdot \underline{S}_k + Q_k = 0$$

$\rightarrow$ collective dissipation depends on local structure of distribution function

$$\Rightarrow \quad \gamma_k = -\frac{Q_k}{W}$$

$\rightarrow$ micro - macro connection

$$= \frac{-\cancel{\omega_k} \frac{\pi \omega_p^2}{k|k|} \frac{\partial f}{\partial v}\Big|_{\omega_k/v}}{\cancel{\omega_k} \frac{\partial \epsilon_r}{\partial \omega}\Big|_k}$$

$$= -\frac{\epsilon_{IM}(k, \omega_k)}{(\partial \epsilon_r / \partial \omega)\big|_{\omega_k}}$$

or

$$\epsilon = \epsilon_r(k,\omega) + i\,\epsilon_{Im}(k,\omega)$$

$$\omega = \omega_k + i\gamma_k \qquad \gamma_k \ll \omega_k$$

$$\epsilon = \epsilon_r(k,\ \omega_k + i\gamma_k) + i\,\epsilon_{Im}(k,\ \omega_k)$$

$$\cong \cancelto{0}{\epsilon_r(k,\omega_k)} + i\gamma_k \left.\frac{\partial\epsilon_R}{\partial\omega}\right|_{\omega_k}$$

$$+\ i\,\epsilon_{Im}(k,\ \omega_k)$$

so

✓

$$\gamma_k = -\epsilon_{Im}(k,\omega_k) \Big/ \left.\frac{\partial\epsilon_R}{\partial\omega}\right|_{\omega_k}$$ agree

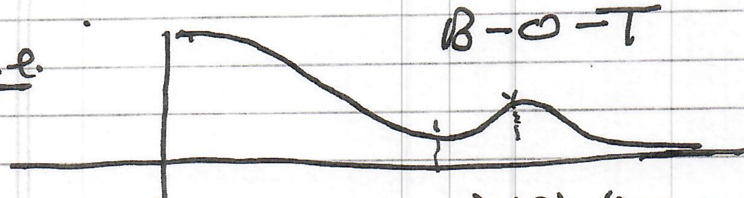
Thus:

- $\partial\langle f\rangle/\partial v < 0$
  $\Rightarrow$ damping

- $\partial\langle f\rangle/\partial v > 0$
  $\Rightarrow$ growth (instability)

i.e.

B-O-T



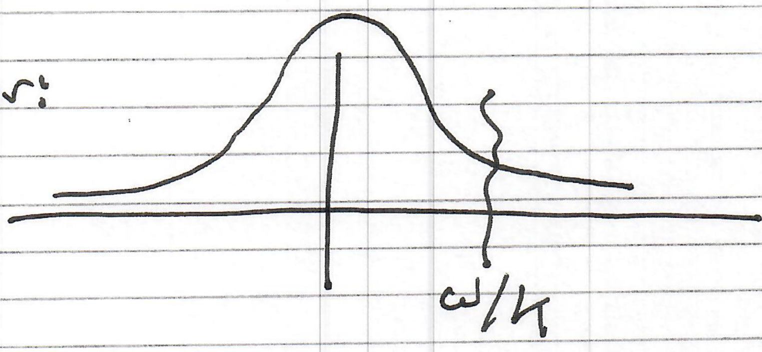
— beam driven

— relate beam + plasma

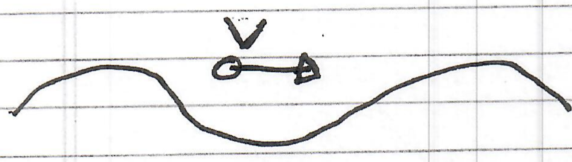
$\partial\langle f\rangle/\partial v > 0$

→ Physics of Landau Damping

Consider:



- Landau damping occurs due to wave-particle resonance at $\omega/k \sim v$
- intuitively, consider wave interaction with a resonant particle



$\omega/k = v_{ph}$

Particle with $v \sim v_{ph}$ 'sees' a DC field

$$\frac{dv}{dt} = \frac{q}{m} E \cos(kx - \omega t)$$

$$= \frac{q}{m} E \cos(k(x - v_{ph} t))$$

if boost to frame at V:

$$x' = x - Vt$$

$$v' = v - V$$

$$a' = a$$

$\Rightarrow$ very heuristic:

$$\frac{dv}{dt} = \frac{q}{m} E \cos\left(k\left(x + (V - v_{ph})t\right)\right)$$

" " — secular (in time) interaction at $v \sim v_{ph}$

— $v \lesssim \omega/k \Rightarrow$ wave does work on particles, loses energy

— $v \gtrsim \omega/k \Rightarrow$ wave does $\ominus$ work, gains energy

Q = # losers − # gainers

$\sim \partial\langle f\rangle/\partial v \big|_{\omega/k}$ (caveat emptor!)

Now, quantitatively:

— $Q = \langle \underline{E}^* \cdot \underline{J} \rangle$

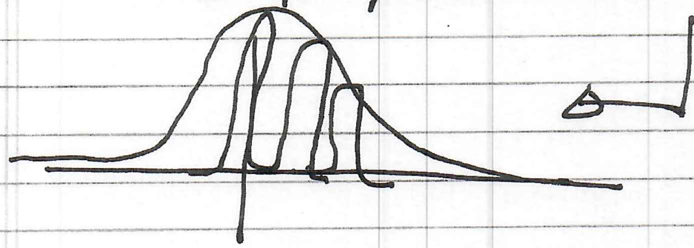
so for beam at V:

$\bar{Q} = \langle q v E \rangle$

$\downarrow$

time avg. dissipated power on resonant beam

Now: - view plasma distribution as superposition of beams



then $Q = \int dv \, \bar{Q}$

$\downarrow$

total dissipation

- Now, calculate $\langle q v E \rangle$:

$v = v_0 + \delta v$ → perturbations induced by wave.

$x = x_0 + \delta v$

so

$$\frac{d}{dt} \delta v = \frac{q}{m} E \Big|_{x_0, v_0}$$

$$\frac{d}{dt} \delta x = \delta v$$

$$\bar{Q} = q\langle v E\rangle$$

$$v = v_0 + \delta v$$

$$E = E(t,\ x = x_0 + \delta x)$$

$$\cong E(t, x_0) + \delta x\ \partial E/\partial x\Big|_{x_0}$$

so, finally: beam power dissipated

$$\bar{Q} = q\ \langle (\underset{\text{osc}}{v_0 + \delta v})\ (\underset{\text{osc}}{E(t, x_0)} + \underset{\text{both osc}}{\delta x\ \partial E/\partial x\Big|_{x_0,t}})\rangle$$

so, retaining quadratic beats:

$$\bar{Q} = q v_0 \langle \delta x\ \partial E/\partial x\Big|_{x_0,t}\rangle$$

$$+ q\ \langle \delta v\ E(t, x_0)\rangle$$

need compute: δx, δv:

$$\frac{d}{dt}\delta v = \frac{q}{m} E(t, x_0)$$

$$x_0 = \underset{\substack{\uparrow \\ \text{unperturbed orbit}}}{x_0'} + v_0 t$$

take:

$x_0' = 0$, convenience

$\omega/k = v_{ph}$

$$E(t, x_0) = \frac{q}{m} E_0 \, e^{\cancelto{1}{ikx_0'}} e^{ik(v_0 - \omega/k)t} e^{\delta t}$$

$$\delta > 0 \quad \text{so} \quad \begin{cases} \delta v \to 0, & t \to -\infty \\ \text{causality} \end{cases}$$

$$\frac{d}{dt} \delta v = \frac{q}{m} E_0 \exp\left(ik\left(v_0 - \omega/k - i\delta\right) t\right)$$

$$\Rightarrow \quad \delta v = \frac{q}{m} E_0 \left. \frac{e^{ik(v_0 - \omega/k - i\delta)t}}{i\left(k(v_0 - v_{ph}) - i\delta\right)} \right|_{-\infty}^{t}$$

$$\delta v = \frac{q}{m} E(t, x_0) \Big/ \left(ik(v_0 - v_{ph}) + \delta\right)$$

and obviously:

$$\delta x = \frac{q}{m} E(t, x_0) \Big/ \left(ik(v_0 - v_{ph}) + \delta\right)^2$$

so

$$\bar{Q} = q v_0 \left\langle \delta x \frac{\partial E}{\partial x} \right\rangle + q \left\langle \delta v \, E \right\rangle$$

$$= q v_0 \left\langle -ik\, E^*(t_0 x_0) \frac{q}{m} \frac{E(t_0 x_0)}{(ik_1(v_0 - v_{ph}) + \delta)^2} \right\rangle$$

$$+ q \left\langle E^*(t_0 x_0) \frac{q}{m} \frac{E(t_0 x_0)}{(ik_1(v_0 - v_{ph}) + \delta)} \right\rangle$$

as $E^* E$ gives DC beat:

$$\bar{Q} = \frac{d}{dv_0} \left\{ \frac{q^2}{2m} \frac{|E|^2 |E|^2 v_0}{[ik_1(v_0 - v_{ph}) + \delta]} \right\}$$

$$= \frac{d}{dv_0} \left\{ \frac{q^2}{2m} |E|^2 \frac{-i v_0}{(k_1(v_0 - v_{ph}) - i\delta)} \right\}$$

real part:

$$\bar{Q} = \frac{d}{dv_0} \left\{ \frac{q^2}{2m} |E|^2 \frac{v_0}{|k_1|} \pi \delta(v_0 - v_{ph}) \right\}$$

Then, for total dissipation, average over ensemble of beams, distributed according to $\langle f \rangle$:

norm to 1

$$Q = n \int dv_0\, \underline{\mathcal{Z}}(v_0) \langle f(v_0) \rangle$$

$$= \int dv_0 \; \langle f(v_0)\rangle \frac{d}{dv_0}\left\{ \frac{n q^2}{2m} |E|^2 \frac{v_0 \pi}{|k|} \delta(v_0 - v_{ph}) \right\}$$

$$= -\pi \frac{\omega_p^2}{|k|} \frac{\omega}{k} \left.\frac{\partial \langle f(v)\rangle}{\partial v}\right|_{\omega/k} \left(|E|^2 / 8\pi \right)$$

$$\boxed{Q = -\pi \frac{\omega_p^2}{|k|} \frac{\omega}{k} \left.\frac{\partial \langle f\rangle}{\partial v}\right|_{\omega/k} \left(|E|^2 / 8\pi \right)}$$

– agrees previous.

* – establishes Landau damping as $\langle E \cdot J \rangle$ work of wave electric field on resonant particles. *

– Fate of energy:

ignoring radiation –

$$\frac{\partial W_k}{\partial t} + \cancel{\nabla \cdot \underline{S}_k} + Q_k = 0$$

$$\partial_t W_k = - Q_k$$

but clearly resonant particles heated:

i.e. will show, in QLT:

$$\partial_t \, RPKED + \partial_t W_k = 0$$

⇒ Landau damping heats resonant piece of distribution at expense of wave energy.

→ BUT:

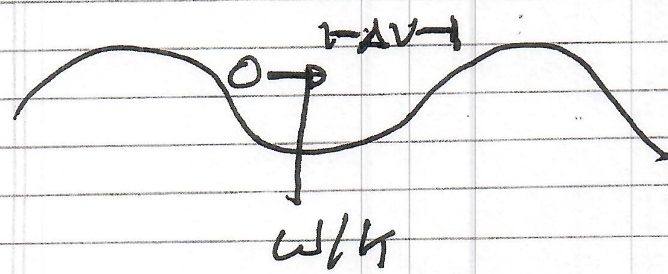
- Landau calculation, and physical argument, are linear ⇒

used linearized, free-streaming unperturbed orbits

- Such linearization valid only for: $t < \tau_b$

↓

bounce time, in wave trough.



i.e. once particle bounces, orbits no longer un-perturbed.

$\Delta v \sim (q\phi/m)^{1/2}$

trapping

$1/\tau_b = k\,\Delta v$

Then $\gamma_k = \gamma_k^{(0)}$; $t < \tau_b$, only

→ Landau resonance forces/drives a picture of plasma as gas of:

- waves + resonant particles
- collective modes as non-resonant particles and fields.
- collective damping via $\langle E \cdot J \rangle$ work on resonant particles.

Remaining:

- How reconcile causality ($\sigma > 0$) and damping ($\gamma < 0$)
  → ivp → see notes.
- Closer look at physics.
  → phase mixing