Pattern Formation in Magnetically Confined Plasmas: Why it Matters

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Physics Colloquium, University of Alberta
May, 2017

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Collaborators: (partial list)

- Yusuke Kosuga → Kyushu Univ., Japan
- Guilhem Dif-Pradalier → CEA, France
- Ozgur Gurcan → Ecole Polytechnique, France
- Arash Ashourvan → PPPL
- Zhibin Guo → UCSD

Magnetically confined plasma → tokamaks

- Nuclear fusion: option for generating large amounts of carbon-free energy – “30 years in the future and always will be… “
- Challenge: ignition -- reaction release more energy than the input energy

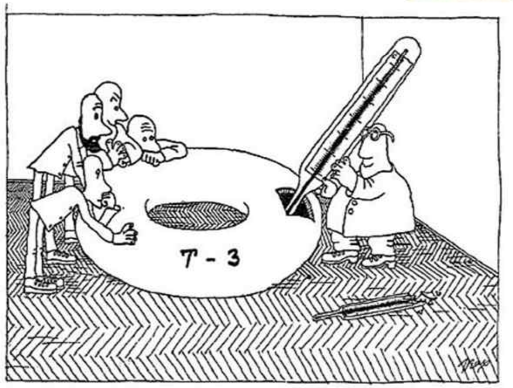
  Lawson criterion:

$$n_i \tau_E T_i > 3\times 10^{21}\mathrm{m}^{-3}\mathrm{s\,keV}$$

  → confinement

  → turbulent transport

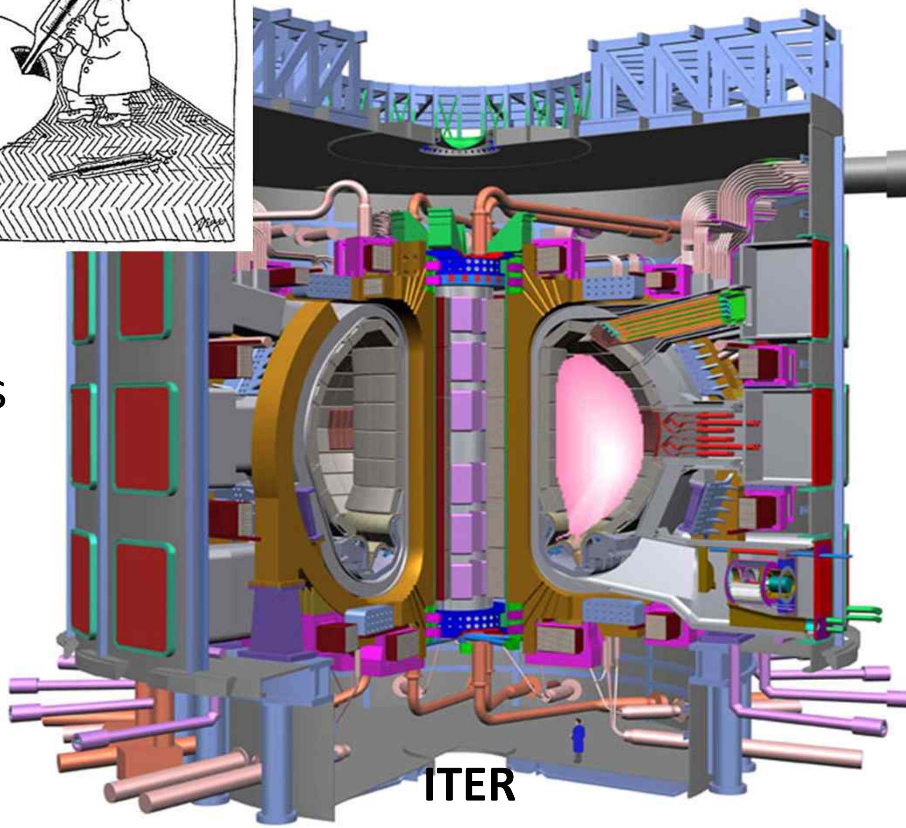
$$\tau_E \sim \frac{W}{P_{in}}$$



- Turbulence: instabilities and collective oscillations

  → low frequency modes dominate the transport ($\omega < \Omega_{ci}$)

- Key problem: Confinement, especially scaling

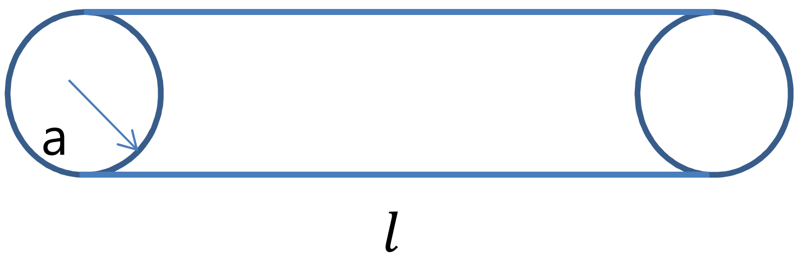


A Simpler Problem:

→ Drag in Turbulent Pipe Flow

- Essence of confinement problem:
  - given device, sources; what profile is achieved?
  - $\tau_E = W/P_{in}$, How optimize W, stored energy
- Related problem: Pipe flow → drag ↔ momentum flux

ΔP → pressure drop



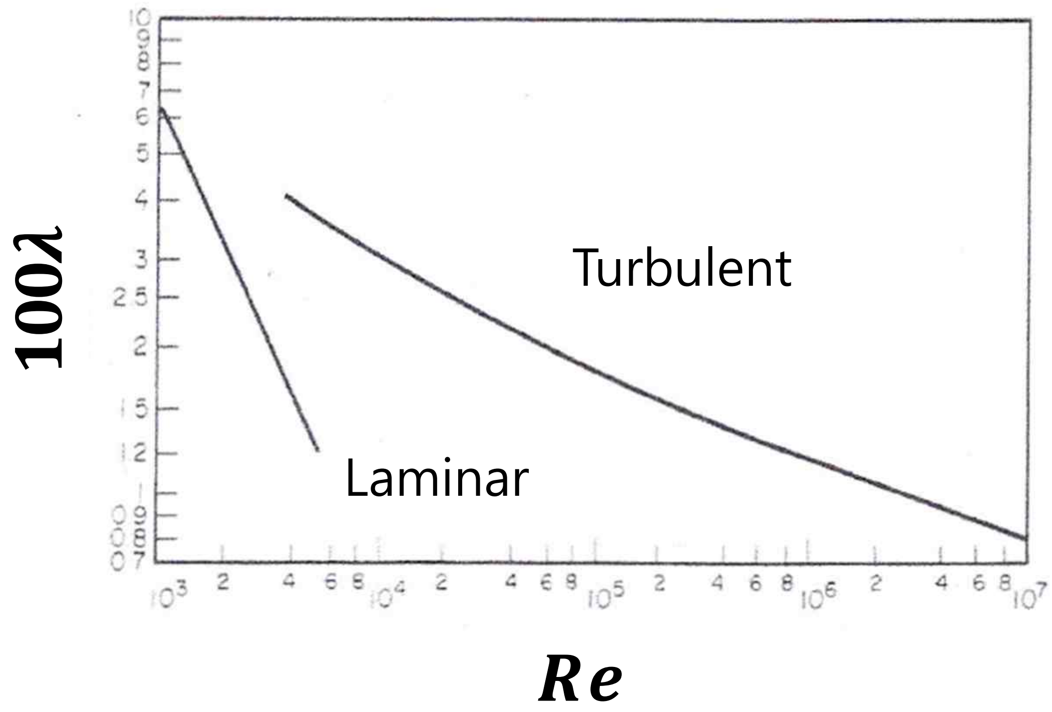
$$\Delta P \pi a^2 = \rho V_*^2 2\pi a l$$

→ friction velocity V_* ↔ u

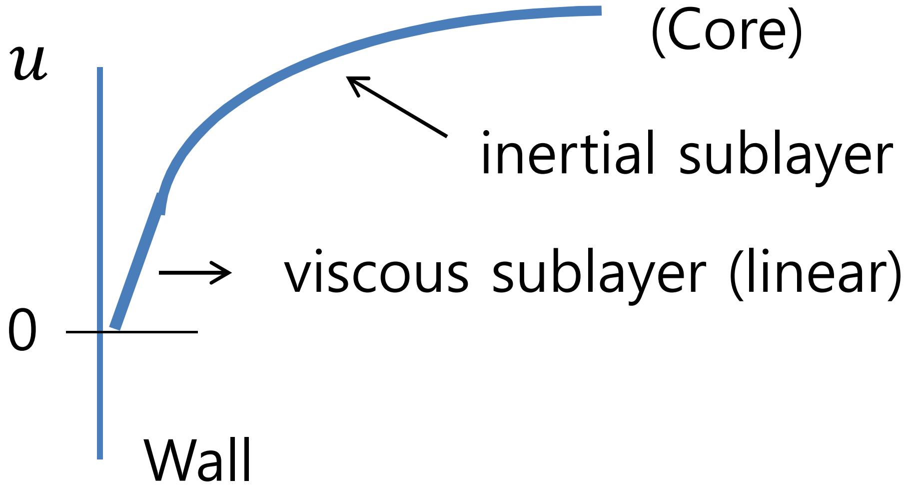
Balance: momentum transport to wall

(Reynolds stress) vs ΔP

➔ Flow velocity profile



$$\lambda = \frac{2a\Delta P/l}{1/2\rho u^2}$$



- <u>Problem</u>: physics of ~ universal logarithmic profile?
- Universality → scale invariance

- <u>Prandtl Mixing Length Theory</u> (1932)
  - Wall stress = $\rho V_*^2 = -\rho \nu_T \, \partial u / \partial x$ or: $\dfrac{\partial u}{\partial x} \sim \dfrac{V_*}{x}$ ← Spatial counterpart of K41

    eddy viscosity

    Scale of velocity gradient?
  - Absence of characteristic scale →

$$\nu_T \sim V_* x$$

$$u \sim V_* \ln(x/x_0)$$

$x \equiv$ mixing length, distance from wall

Analogy with kinetic theory …

$\nu_T = \nu \rightarrow x_0$, viscous layer → $x_0 = \nu / V_*$

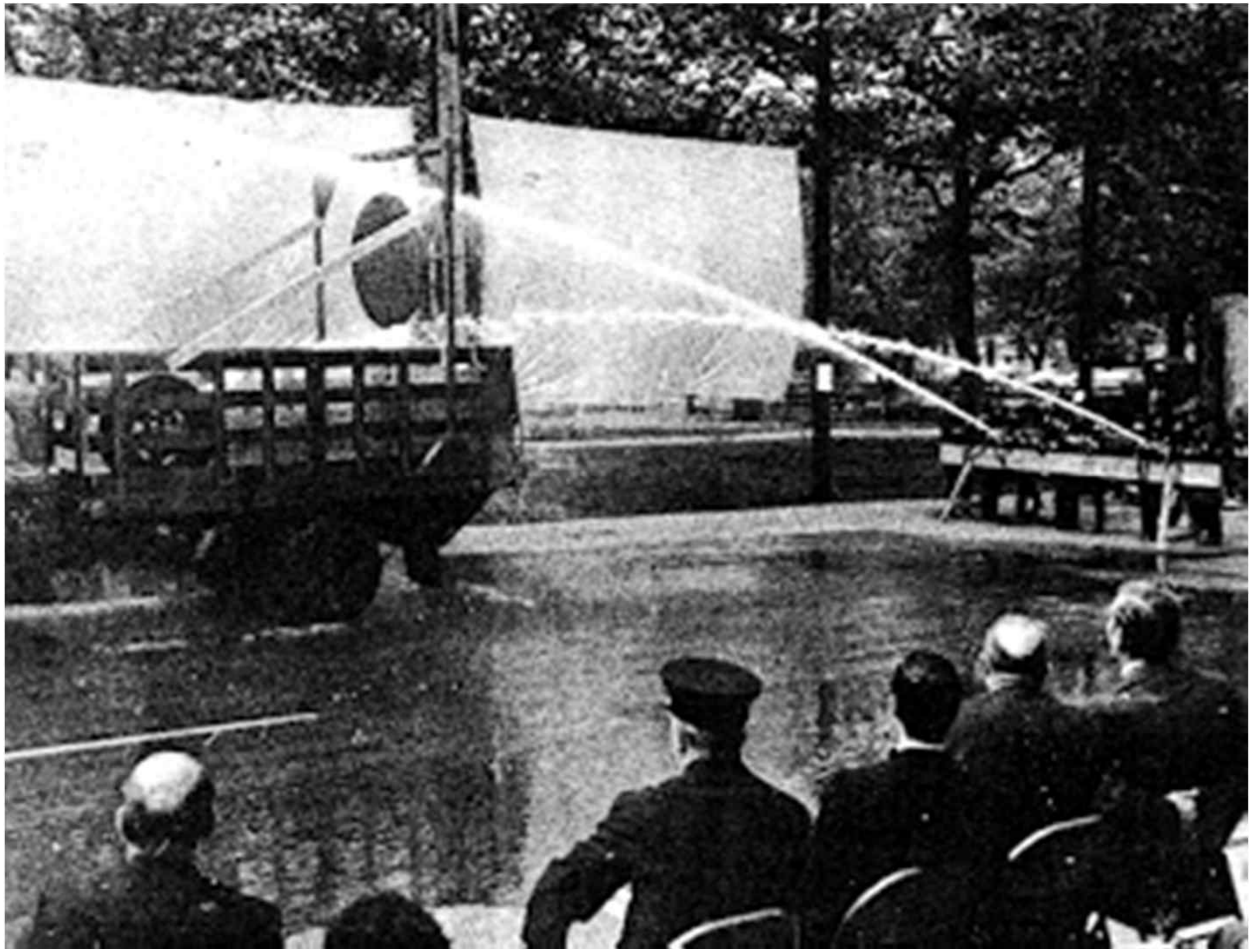
Some key elements:

- Momentum flux driven process
- Turbulent diffusion model of transport - eddy viscosity
- Mixing length – scale selection

  $\sim x$ → macroscopic, eddys span system $x_0 < x < a$

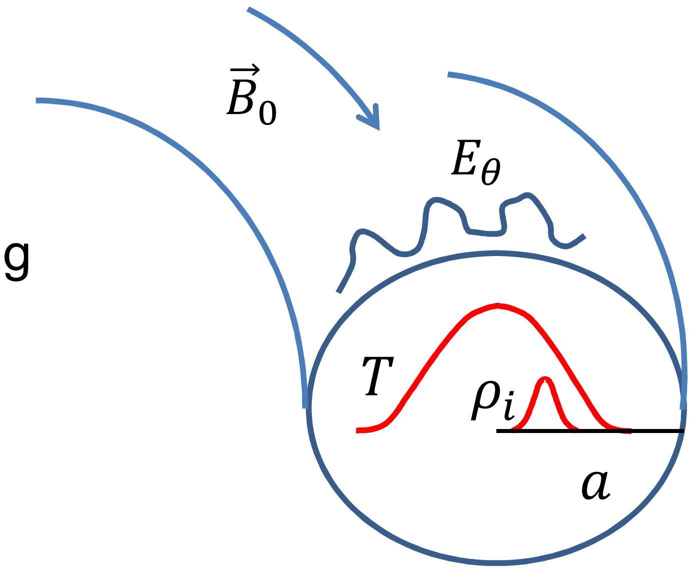
  → ~ flat profile – strong mixing

- Self-similarity → x ↔ no scale, within $[x_0, a]$
- Reduce drag by creation of buffer layer i.e. steeper gradient than inertial sublayer (by polymer) – enhanced confinement



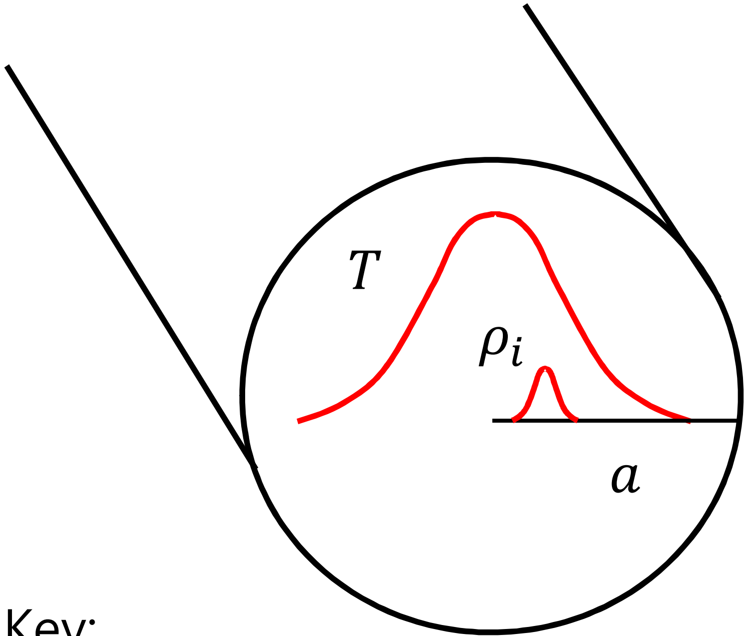
Without vs With Polymers
Comparison → NYFD 1969

Primer on Turbulence in Tokamaks I



- Strongly magnetized
  - Quasi 2D cells, Low Rossby #
- * – Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance) - pinning
- $\vec{V}_\perp = +\frac{c}{B}\,\vec{E} \times \hat{z}, \quad \frac{V_\perp}{l\Omega_{ci}} \sim R_0 \ll 1$
- $\nabla T_e, \nabla T_i, \nabla n$ driven
- Akin to thermal convection with: g → magnetic curvature
- → $\mathrm{Re} \approx VL/\nu$ ill defined, not representative of dynamics
- Resembles wave turbulence, not high Re Navier-Stokes turbulence
- → $K \sim \tilde{V}\tau_c/\Delta \sim 1$ → *Kubo #* ≈ 1
- → Broad dynamic range, due electron and ion scales, i.e. a, ρ_i, ρ_e

Primer on Turbulence in Tokamaks II



Key:

2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

$\rho_* \equiv \rho/a$ ➔ key ratio

$\rho_* \ll 1$

- Characteristic scale ~ few ρ_i → "mixing length"
- Characteristic velocity $v_d \sim \rho_* c_s$
- Transport scaling: $D_{GB} \sim \rho\, V_d \sim \rho_*\, D_B$

$$D_B \sim \rho\, c_s \sim T/B$$

- i.e. Bigger is better! ➔ sets profile scale via heat balance (Why ITER is huge…)
- Reality: $D \sim \rho_*^{\alpha}\, D_B$, $\alpha < 1$ ➔ 'Gyro-Bohm breaking'
- 2 Scales, $\rho_* \ll 1$ ➔ key contrast to pipe flow

THE Question ↔ Scale Selection

- Expectation (from pipe flow):
  - $l \sim a$
  - $D \sim D_B$
- Hope (mode scales)
  - $l \sim \rho_i$
  - $D \sim D_{GB} \sim \rho_* D_B$
- Reality: $D \sim \rho_*^{\alpha} D_B, \quad \alpha < 1$

Why? What physics competition set α?

The System Fundamentals: $R_0 \ll 1$ Fluids $(\Omega \leftrightarrow \Omega_i)$

- Kelvin's Theorem for rotating system

$$\omega \to \omega + 2\mathbf{\Omega}$$

relative planetary

$$\oint \mathbf{v} \cdot d\mathbf{l} = \int d\mathbf{a} \cdot (\omega + 2\Omega) \equiv C$$

$$\dot{C} = 0$$

- $Ro = V/(2\Omega L) \ll 1 \quad \to \quad \mathbf{V} \cong -\nabla_{\perp} p \times \hat{z}/(2\Omega)$ geostrophic balance

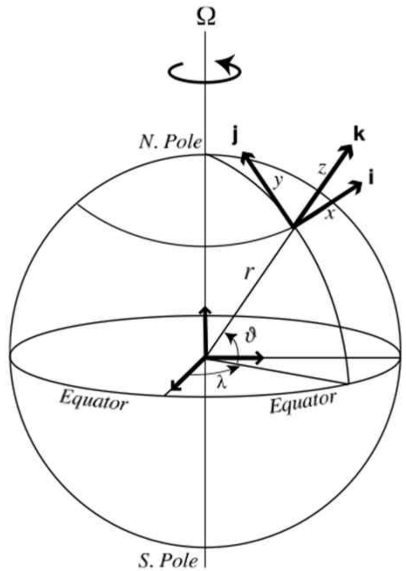
$\to$ 2D dynamics

- Displacement on beta plane

$$\dot{C} = 0 \quad \to \quad \frac{d}{dt}\omega \cong -\frac{2\Omega}{A}\sin\theta_0 \frac{dA}{dt}$$

$$= -2\Omega \frac{d\theta}{dt} = -\beta V_y$$

$$\omega = \nabla^2 \phi \qquad \beta = 2\Omega \sin\theta_0 / R$$



Fundamentals II

- Q.G. equation $\frac{d}{dt}(\omega + \beta y) = 0$

n.b. topography

- Locally Conserved PV $q = \omega + \beta y$

$q = \omega / H + \beta y$

- Latitudinal displacement $\longrightarrow$ change in relative vorticity
- Linear consequence $\longrightarrow$ Rossby Wave

$\omega = -\beta k_x / k^2$

$\omega = 0$ → zonal flow

observe: $v_{g,y} = 2\beta \underline{k_x k_y} / (k^2)^2$

→ Rossby wave intimately connected to momentum transport

- Latitudinal PV Flux $\longrightarrow$ circulation

→ Isn't this Talk re: Plasma?

→ 2 Simple Models a.) Hasegawa-Wakatani (collisional drift inst.)

b.) Hasegawa-Mima (DW)

a.) $\mathbf{V} = \frac{c}{B}\hat{z} \times \nabla\phi + \underset{\rightarrow m_s}{\mathbf{V}_{pol}}$

$$L > \lambda_D \rightarrow \nabla \cdot \mathbf{J} = 0 \rightarrow \nabla_\perp \cdot \mathbf{J}_\perp = -\nabla_\| J_\|$$

$$J_\perp = n|e|V_{pol}^{(i)}$$

$$J_\| : \eta J_\| = -\cancel{(1/c)\partial_t A_\|} - \nabla_\| \phi + \nabla_\| p_e$$

e.s.

b.) $dn_e/dt = 0$

$$\rightarrow \quad \frac{dn_e}{dt} + \frac{\nabla_\| J_\|}{-n_0|e|} = 0$$

n.b.

MHD: $\partial_t A_\|$ v.s. $\nabla_\| \phi$

DW: $\nabla_\| p_e$ v.s. $\nabla_\| \phi$

So H-W

$$\rho_s^2 \frac{d}{dt} \nabla^2 \hat{\phi} = -D_\parallel \nabla_\parallel^2 (\hat{\phi} - \hat{n}/n_0) + \nu \nabla^2 \nabla^2 \hat{\phi}$$

$$\frac{d}{dt} n - D_0 \nabla^2 \hat{n} = -D_\parallel \nabla_\parallel^2 (\hat{\phi} - \hat{n}/n_0)$$

$D_\parallel k_\parallel^2 / \omega$

is key parameter

$\rightarrow \langle \tilde{v}_r \tilde{n} \rangle \neq 0$

and instability

b.) $D_\parallel k_\parallel^2 / \omega \gg 1 \rightarrow \hat{n}/n_0 \sim e\hat{\phi}/T_e \qquad (m, n \neq 0)$

$$\frac{d}{dt} (\phi - \rho_s^2 \nabla^2 \phi) + v_* \partial_y \phi = 0$$

$\rightarrow$ H-M

n.b. $\mathrm{PV} = \phi - \rho_s^2 \nabla^2 \phi + \ln n_0(x) \qquad \frac{d}{dt} (\mathrm{PV}) = 0$

An infinity of technical models follows …

III) Patterns in Turbulence

→ Avalanches

→ Zonal Flows

➔ Spatial structure of turbulence profile

➔ Pattern selection competition

→ "Truth is never pure and rarely simple" (Oscar Wilde)

Transport: Local or Non-local?

- 40 years of fusion plasma modeling
  - local, diffusive transport

$$Q = -n\chi(r)\nabla T, \quad \chi \leftrightarrow D_{GB}$$

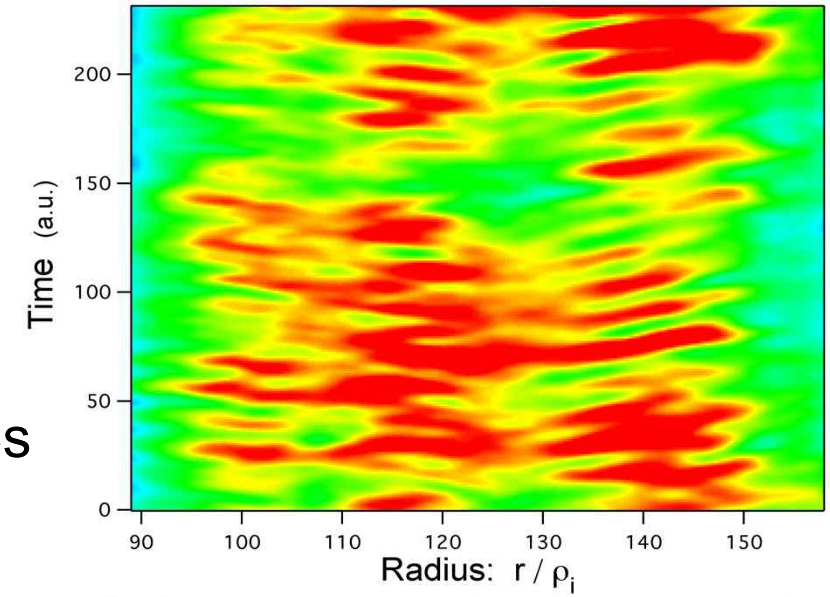
- 1995 → increasing evidence for:
  - transport by <u>avalanches</u>, as in sand pile/SOCs
  - turbulence propagation and invasion fronts
  - "non-locality of transport"

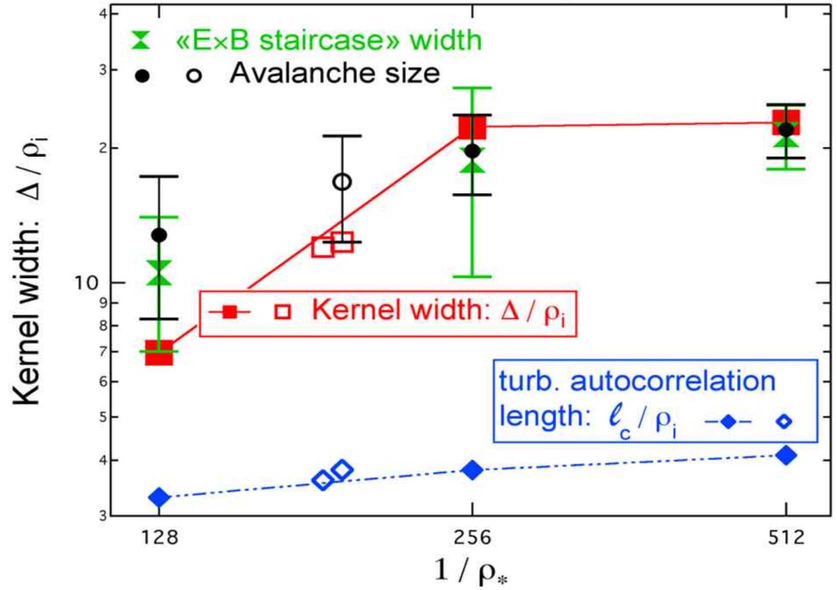
$$Q = -\int \kappa(r,r')\nabla T(r')dr'$$

$$\kappa(r,r') \sim S_0 / \left[(r-r')^2 + \Delta^2\right]$$

- Physics:
  - Levy flights, SOC, turbulence fronts…
- Fusion:
  - gyro-Bohm breaking
    (ITER: significant ρ_* extension)

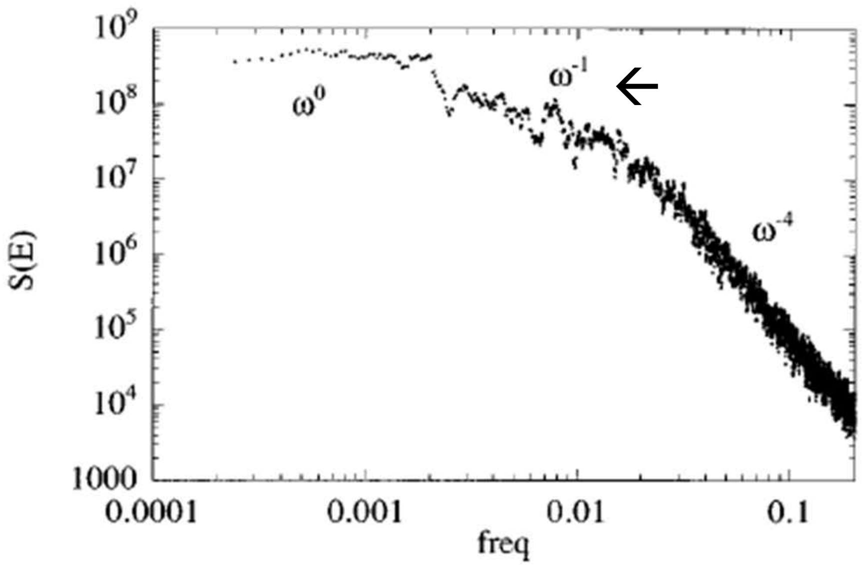
→ *fundamentals of turbulent transport modeling??*



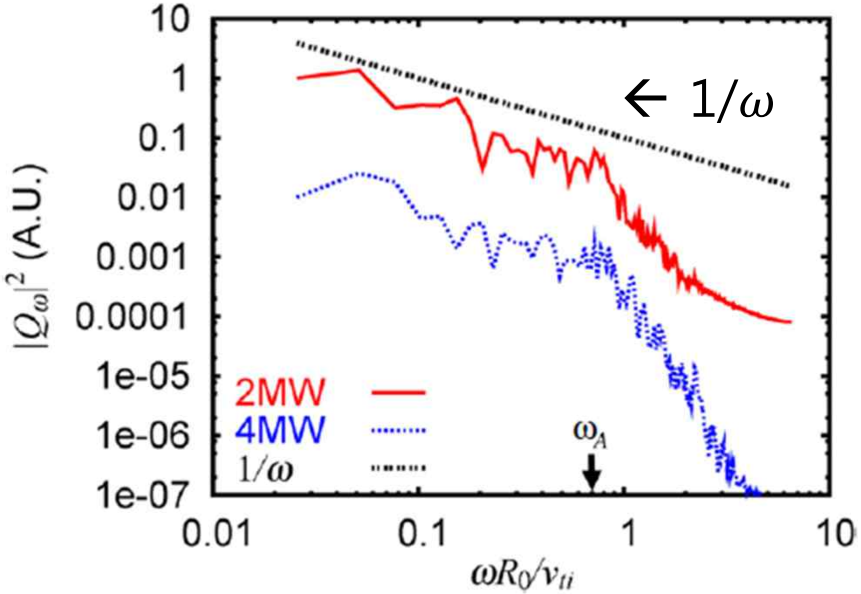


Dif-Pradalier et al. 2010

- ‘Avalanches’ form! – flux drive + geometrical ‘pinning’



Newman PoP96 (sandpile)
(Autopower frequency spectrum of ‘flip’)

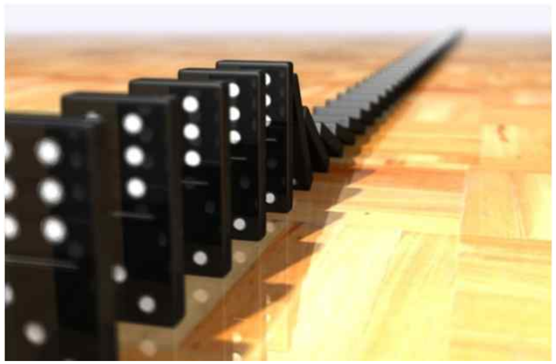


GK simulation also exhibits avalanching
(Heat Flux Spectrum) (Idomura NF09)

- Avalanching is a likely cause of ‘gyro-Bohm breaking’ → Intermittent Bursts

➔ localized cells self-organize to form transient, extended transport events

- Akin domino toppling:
- Natural route to scale invariance on $[a, \Delta_c \sim \rho_i]$



Toppling front can penetrate beyond region of local stability

Origin:

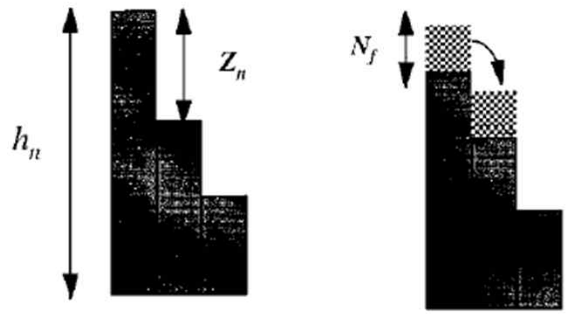
- Cells "pinned" by magnetic geometry → resonances
- Remarkable

  Similarity:

TABLE I. Analogies between the sandpile transport model and a turbulent transport model.

Turbulent transport in toroidal plasmas	Sandpile model
Localized fluctuation (eddy)	Grid site (cell)
Local turbulence mechanism:	*Automata rules*:
Critical gradient for local instability	Critical sandpile slope (Z_{crit})
Local eddy-induced transport	Number of grains moved if unstable (N_f)
Total energy/particle content	Total number of grains (total mass)
Heating noise/background fluctuations	Random rain of grains
Energy/particle flux	Sand flux
Mean temperature/density profiles	Average slope of sandpile
Transport event	Avalanche
Sheared electric field	Sheared flow (sheared wind)

Automaton toppling
↔ Cell/eddy overturning



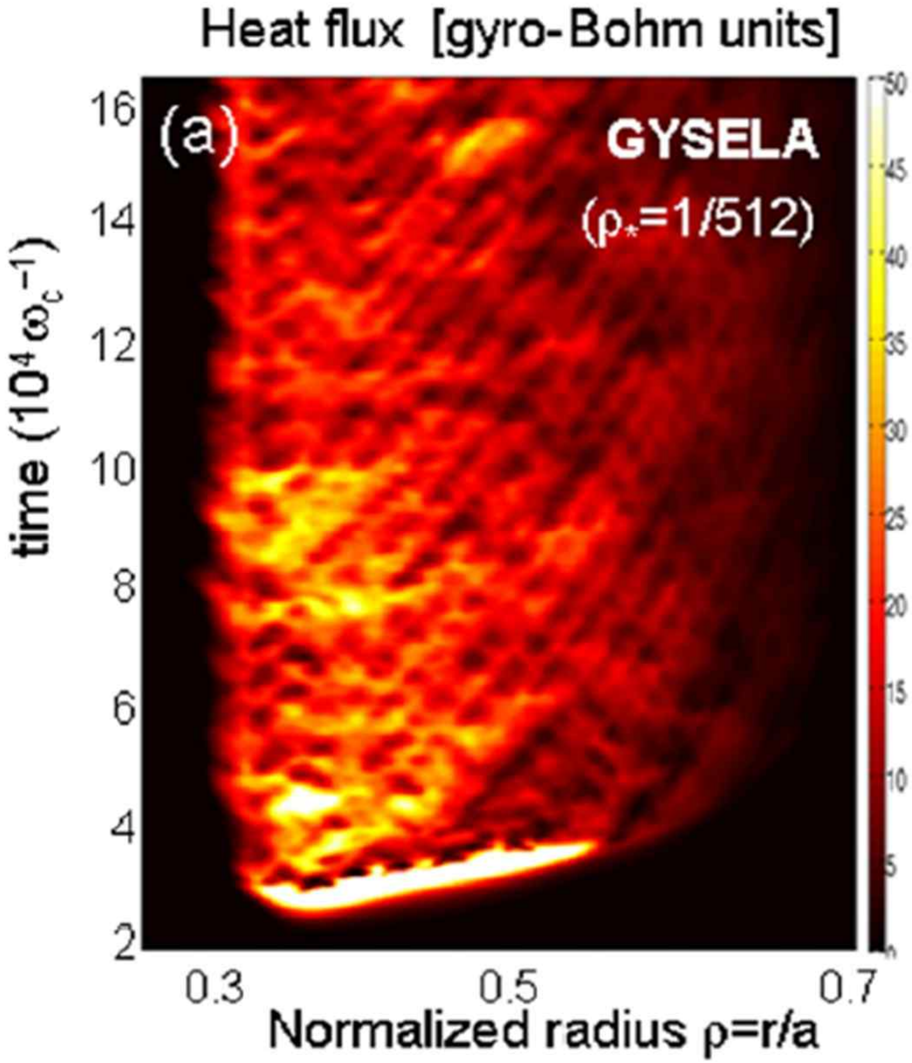
and can cooperate!

→ Avalanches happen!

FIG. 1. A cartoon representation of the simple cellular automata rules used to model the sandpile.

- GYSELA Simulation Results: Avalanches Do 'matter'

GYSELA, rhostar=1/512 [Sarazin et al., NF 51 (2011) 103023]



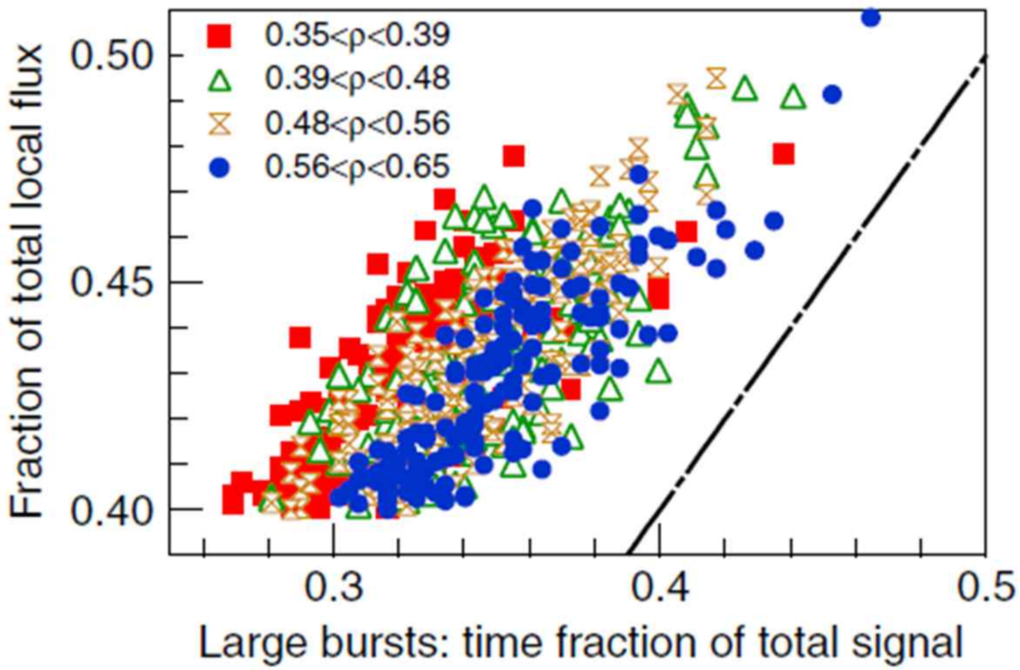


Figure 2. Fraction of the local radial turbulent heat flux carried out by a certain fraction of the largest scale bursts, as estimated from figure 1(a) (GYSELA data). Each point refers to one specific radial location. The colours allow one to distinguish four different radial domains. The considered time series ranges from $\omega_{c0}t = 56\,000$ to $\omega_{c0}t = 163\,000$.

- Distribution of Flux Excursion and Shear Variation

GYSELA, rhostar=1/64 [Sarazin et al., NF 50 (2010) 054004]

Asymmetry!

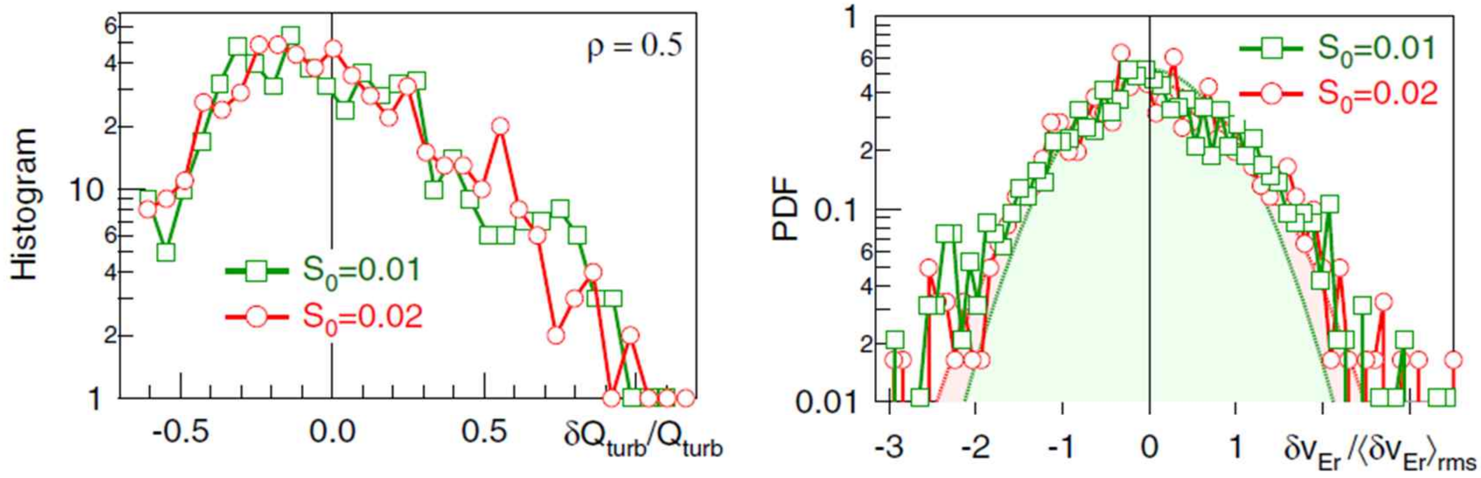


Figure 7. (Left) histogram of the turbulent heat flux Q_{turb} at $\rho = 0.5$ for two magnitudes of the source ($\rho_* = 1/64$). δQ_{turb} stands for the difference between Q_{turb} and its time average, taken over the entire non-linear saturation phase. (Right) corresponding PDF of the fluctuations of the radial component of the electric drift. (Colour online.)

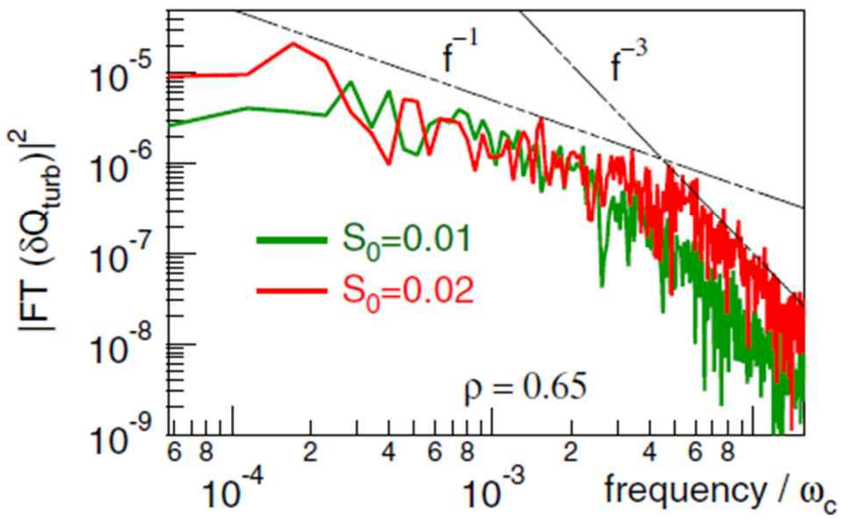
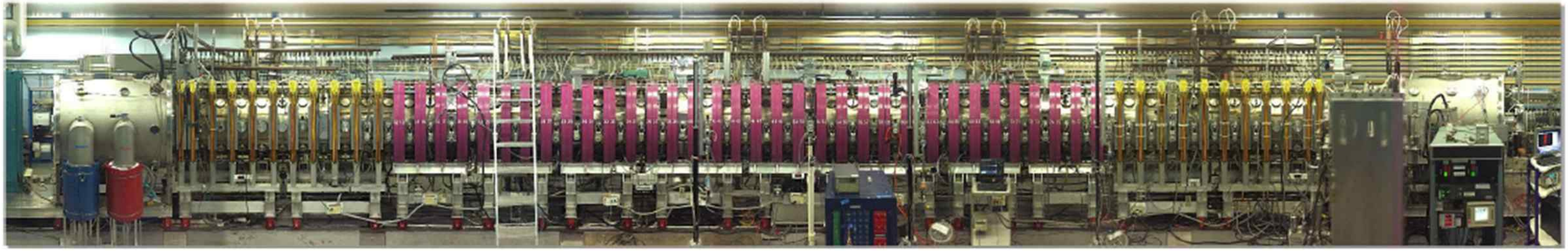
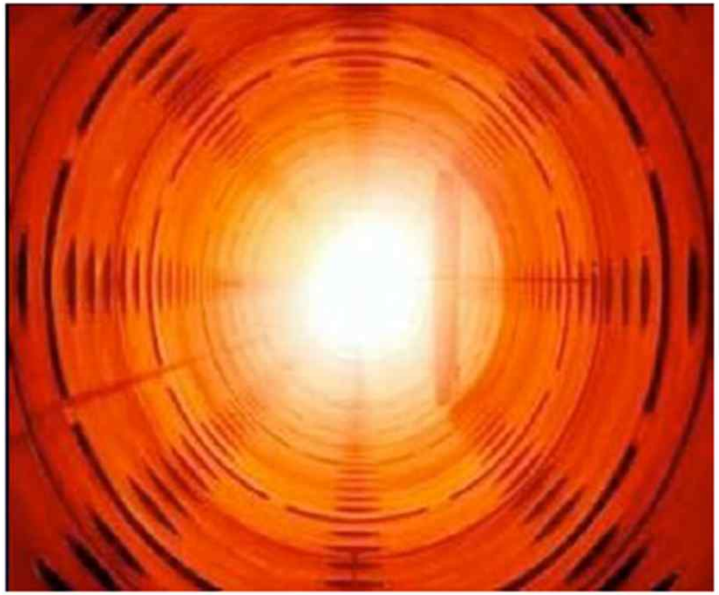


Figure 8. Frequency Fourier spectrum of the turbulent heat flux at $\rho = 0.65$ for two magnitudes of the source ($\rho_* = 1/64$). (Colour

Large Plasma Device



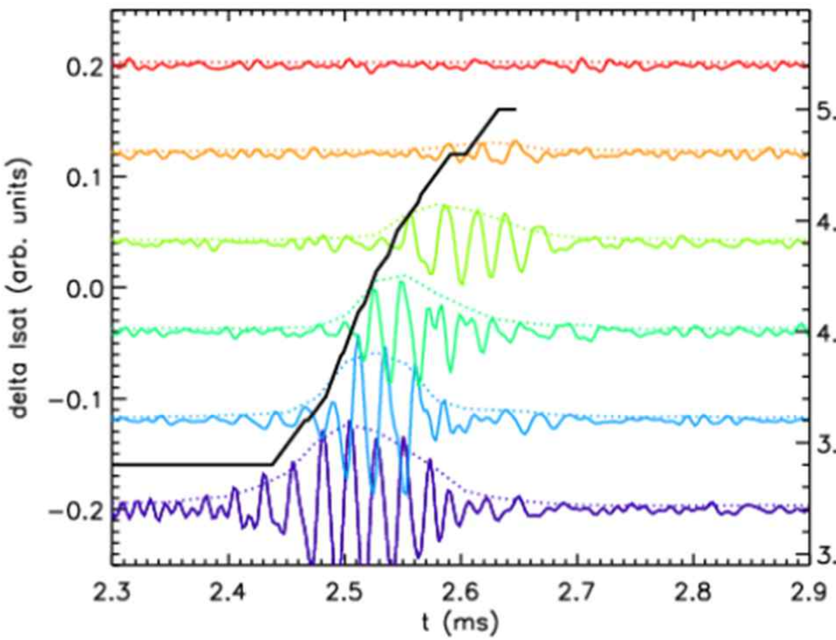


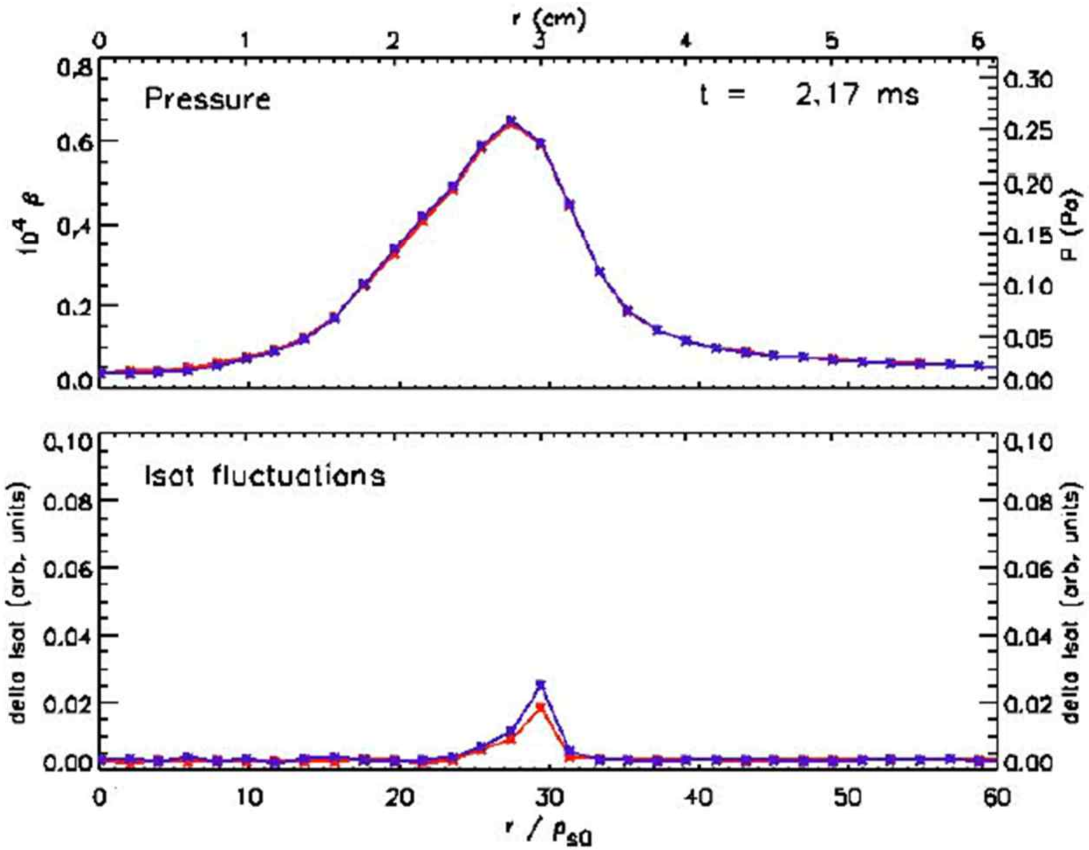
- Helium plasma
- $B_0 = 1000$ G
- $n_e \approx 10^{12}$ cm^{-3}
- $\beta_e = 10^{-4}$

Basic Experiments on Avalanching: Compernolle, Sydora, et. al.

Outer avalanche: drift wave dynamics

- Avalanche onset after fast growth of drift waves
- Avalanche carried radially by drift waves





Black line: position of steepest gradient

See talk by R. Sydora, CO5.00003, Monday 2:24 PM

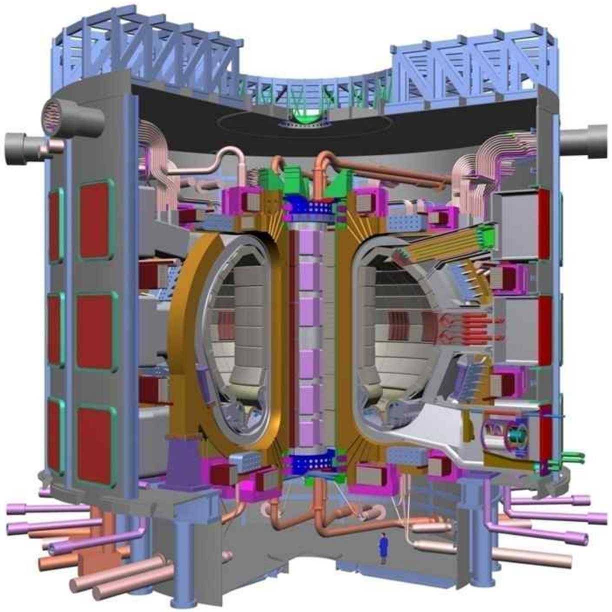
Avalanche propagation observed

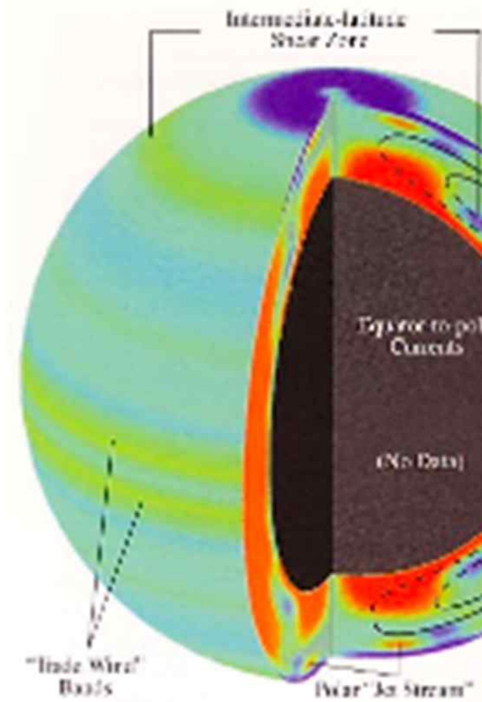
But: Shear Flows Also ‘Natural’ to Tokamaks

- Zonal Flows Ubiquitous for:
  - ~ 2D fluids / plasmas $R_0 < 1$

    Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification

    Ex: MFE devices, giant planets, stars…





The sun

Shear Flows !? – Significance?

No shear

shear

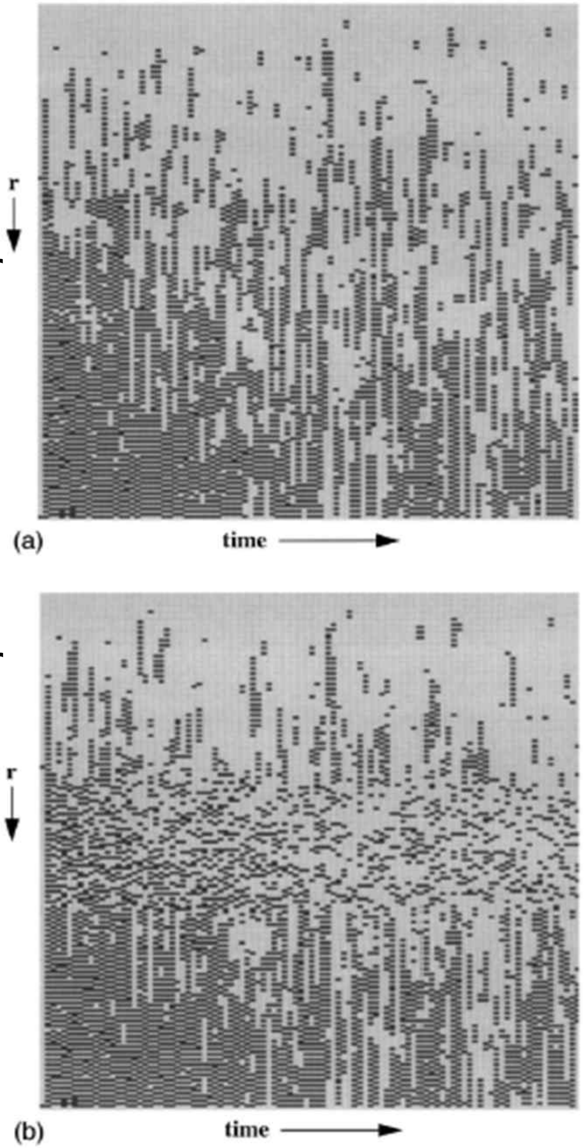


FIG. 11. Time evolution of the overturning sites (like Fig. 4). The avalanches do not appear continous in time because only every 50th time step is shown. (a) The shear-free case shows avalanches of all lengths over the entire radius. (b) The case with sheared flow shows the coherent avalanches being decorrelated in the shear zone in the middle of the pile.

How is transport affected?

➔ shear decorrelation!

Back to sandpile model:

2D pile +

sheared flow of

grains

Shearing flow
decorrelates
Toppling sequence

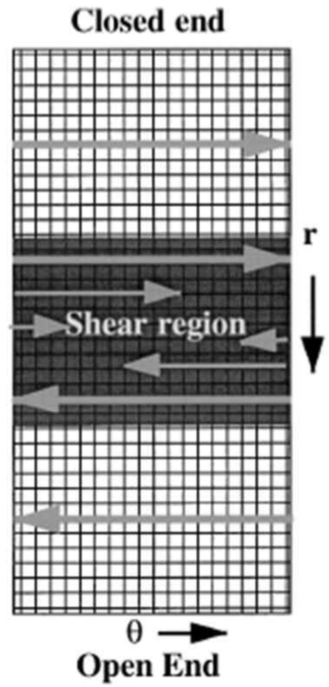
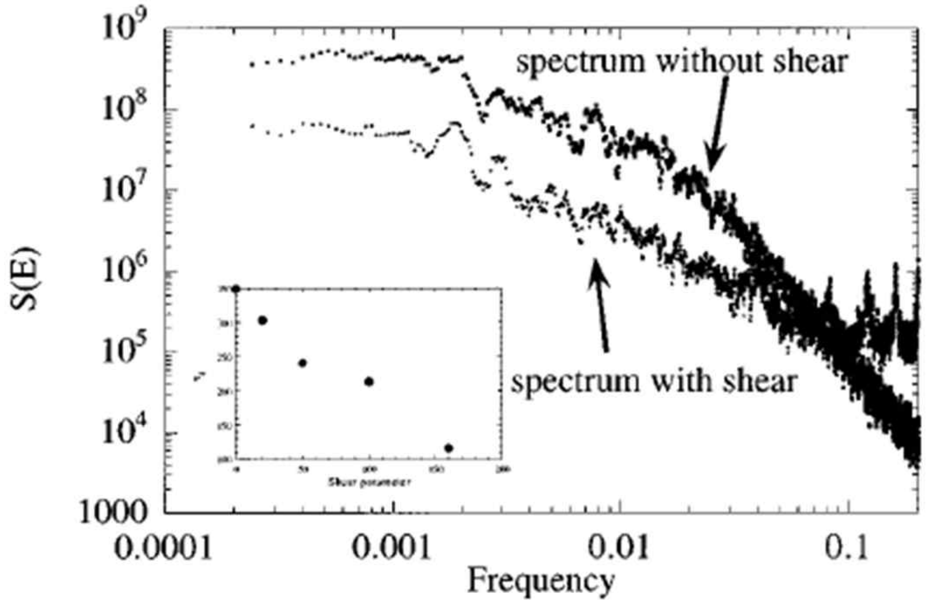


FIG. 10. A cartoon of the sandpile with a shear flow zone. The whole pile is flowing to the right at the top and to the left at the bottom connected by a variable sized region of sheared flow.

Avalanche coherence destroyed by shear flow

- # Implications:

Spectrum of Avalanches



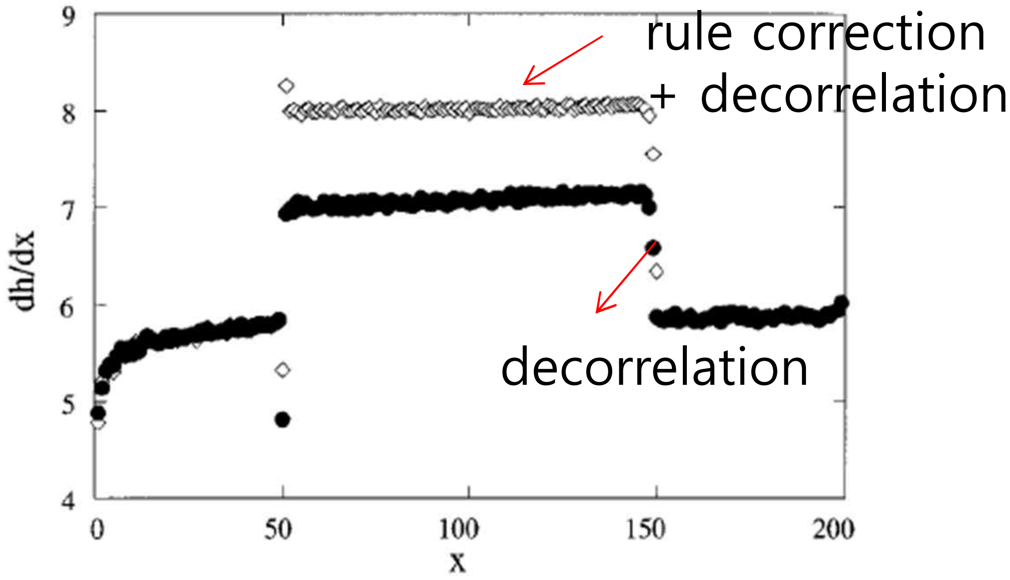
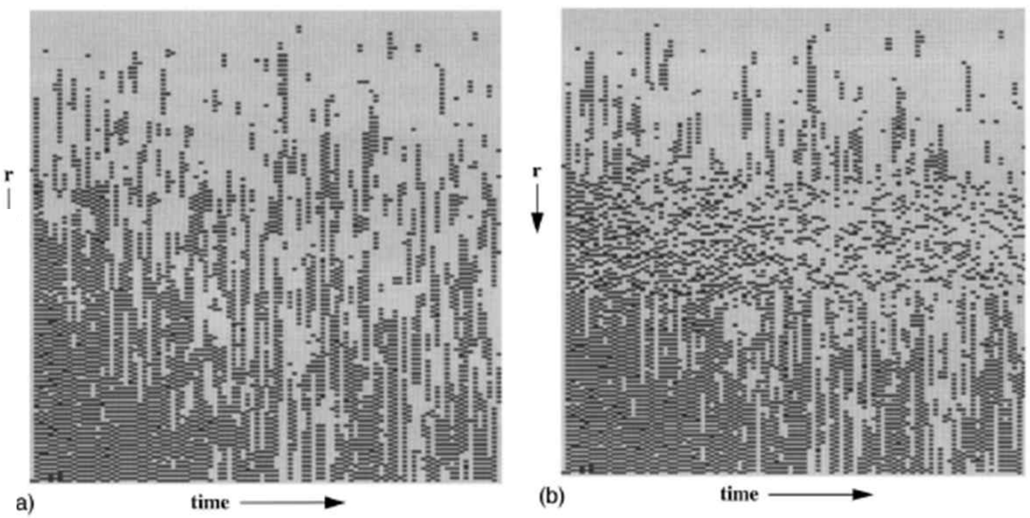


FIG. 14. The slopes of a sandpile with a shear region in the middle, including all the shear effects (diamonds) and just the transport decorrelation and the linear effect (circles).



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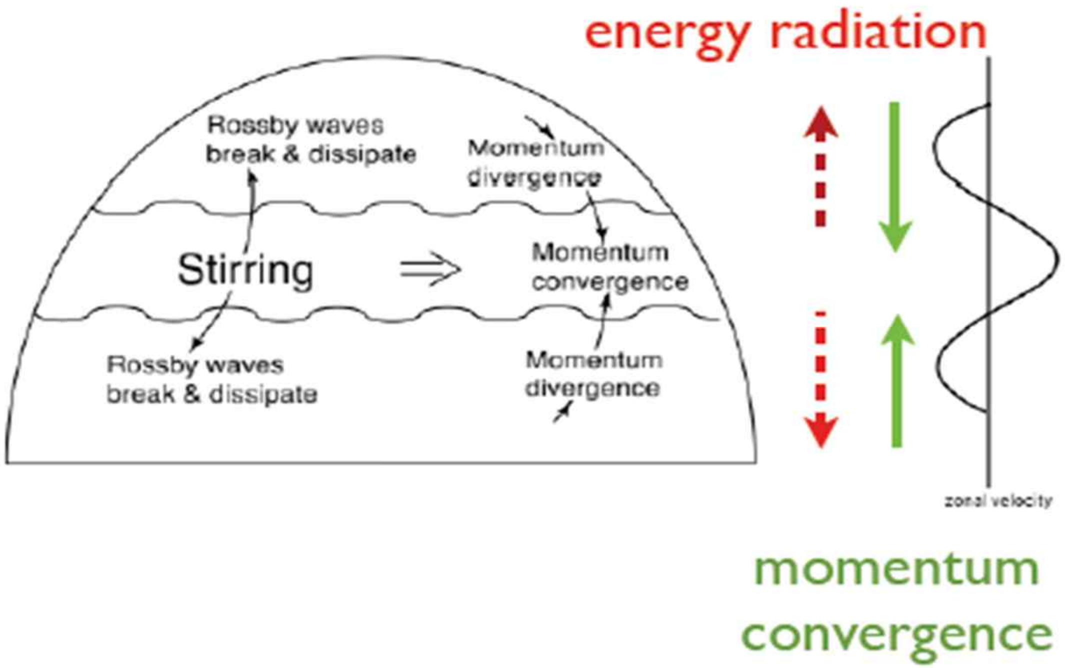
N.B.

- Profile steepens for <u>unchanged</u> toppling rules
- <u>Distribution</u> of avalanches changed

→ How do Zonal Flow Form?

Simple Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:



Rossby Wave:

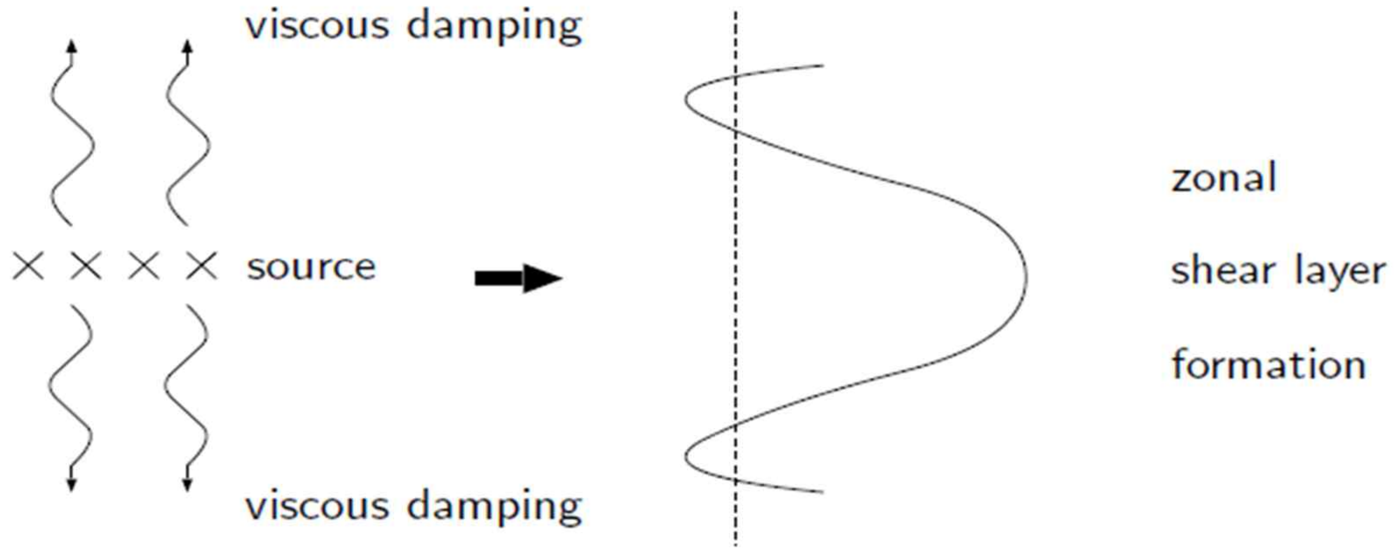
$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

$$v_{gy} = 2\beta \frac{k_x k_y}{(k_\perp^2)^2}, \ \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_{\vec{k}} -k_x k_y \left|\hat{\phi}_{\vec{k}}\right|^2$$

$\therefore v_{gy} v_{phy} < 0$ → Backward wave!

→ Momentum convergence at stirring location

- …"the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves $\Rightarrow$ incoming wave momentum flux

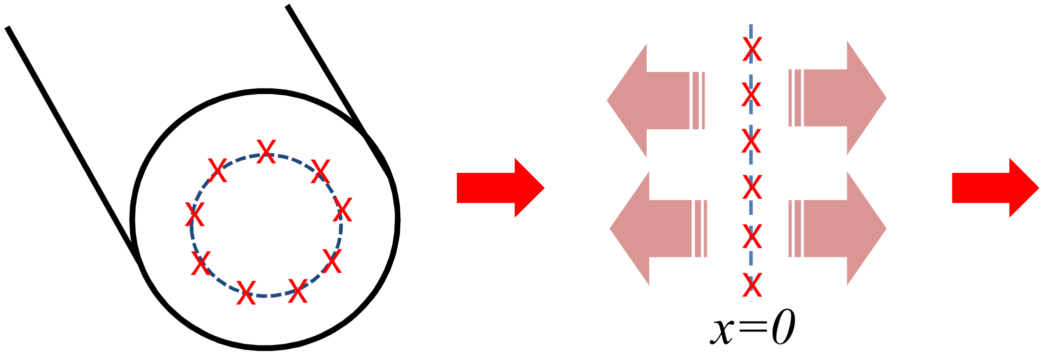


- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena
    → Both 'negative diffusion' phenomena

Wave-Flows in Plasmas

MFE perspective on Wave Transport in DW Turbulence

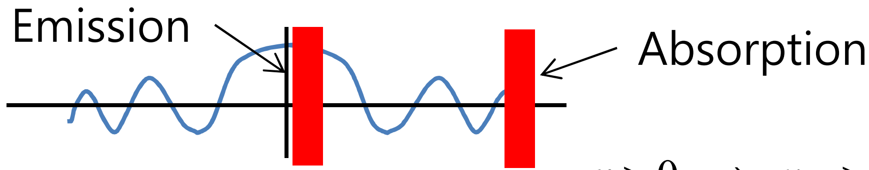
- localized source/instability drive intrinsic to drift wave structure



$x=0$

radial structure

– couple to damping ↔ outgoing wave



– $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$

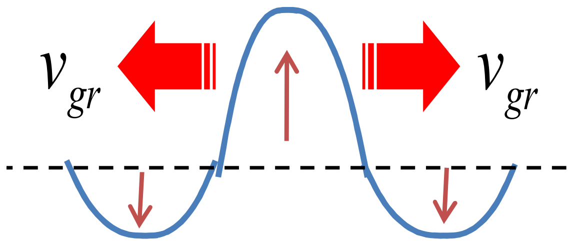
$\langle v_{rE} v_{\theta E} \rangle = -\dfrac{c^2}{B^2} |\phi_{\vec{k}}|^2 \, k_r k_\theta < 0$

$x > 0 \;\Rightarrow\; v_{gr} > 0$

$x < 0 \;\Rightarrow\; v_{gr} < 0$

$v_* < 0 \;\Rightarrow\; k_r k_\theta > 0$

- outgoing wave energy flux → incoming wave momentum flux
  → counter flow spin-up!



- zonal flow layers form at excitation regions

Plasma Zonal Flows I

- What is a Zonal Flow? – Description?
  - *n = 0* potential mode; *m = 0* (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport (*n = 0*)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

i.e. ZF's soak up turbulence energy

Plasma Zonal Flows II

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    $\rightarrow$ Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- Charge Balance $\rightarrow$ polarization charge flux $\rightarrow$ Reynolds force
  - Polarization charge ➡ $-\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$

    polarization length scale *ion GC* *electron density*

  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2 \left\langle \tilde{v}_{rE} \nabla_\perp^2 \tilde{\phi} \right\rangle \neq 0$ ⬌ 'PV transport'

    polarization flux $\rightarrow$ What sets cross-phase?

  - If 1 direction of symmetry (or near symmetry):

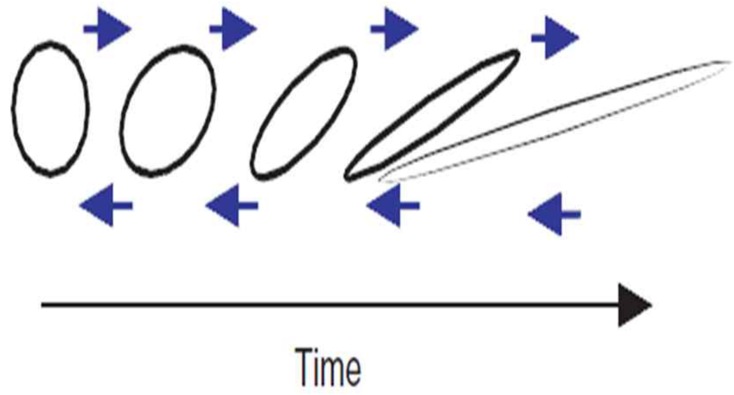
    $-\rho^2 \left\langle \tilde{v}_{rE} \nabla_\perp^2 \tilde{\phi} \right\rangle = -\partial_r \left\langle \tilde{v}_{rE} \tilde{v}_{\perp E} \right\rangle$ (Taylor, 1915)

    $-\partial_r \left\langle \tilde{v}_{rE} \tilde{v}_{\perp E} \right\rangle$ ➡ Reynolds force ➡ Flow

Zonal Flows Shear Eddys I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree'66, BDT'90)
  - radial scattering + $\langle V_E \rangle' \rightarrow$ hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$

  ➔ <u>shearing restricts mixing scale!</u>



- Other shearing effects (linear):

  Response shift
  and dispersion

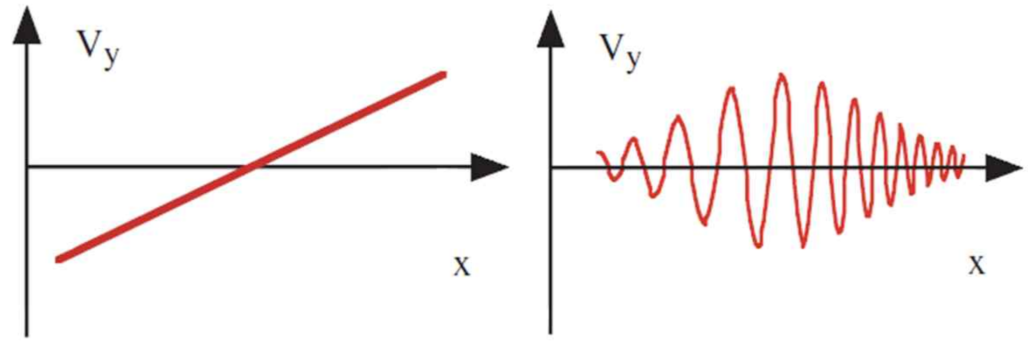
  - spatial resonance dispersion: $\omega - k_{\|} v_{\|} \Rightarrow \omega - k_{\|} v_{\|} - k_\theta \langle V_E \rangle' (r - r_0)$
  - differential response rotation → especially for kinetic curvature effects

Shearing II – Eddy Population

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. '98, et. seq.)

  Coherent interaction approach (L. Chen et. al.)

- $dk_r / dt = -\partial(\omega + k_\theta V_E)/\partial r \,;\; V_E = \langle V_E \rangle + \widetilde{V}_E$



Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

Zonal Random shearing $: \langle \delta k_r^2 \rangle = D_k \tau$

$$D_k = \sum_q k_\theta^2 \left| \widetilde{V}'_{E,q} \right|^2 \tau_{k,q}$$

- Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

→ Evolves population in response to shearing

Shearing III

- Energetics: Books must Balance for Reynolds Stress-Driven Flows!
- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle\right) \Rightarrow \frac{\partial}{\partial t}\langle \varepsilon\rangle = -\int d\vec{k}\, V_{gr}(\vec{k}) D_{\vec{k}} \frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr} = \frac{-2k_r k_\theta V_* \rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle / dk_r < 0$, Z.F. shearing damps wave energy

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t \delta V_\theta + \partial\left(\delta\langle \tilde{V}_r \tilde{V}_\theta\rangle\right)/\partial r = \gamma \delta V_\theta$$

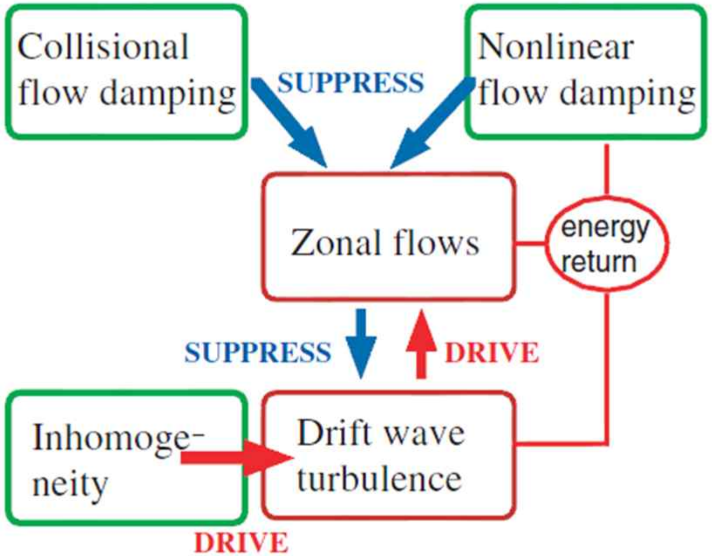
$$\delta\langle \tilde{V}_r \tilde{V}_\theta\rangle \sim \frac{k_r k_\theta \delta N}{(1+k_\perp^2\rho_s^2)^2}$$

N.B.: Wave decorrelation essential: Equivalent to PV transport (c.f. Gurcan et. al. 2010)

- Bottom Line:
  - Z.F. growth due to shearing of waves
  - "Reynolds work" and "flow shearing" as relabeling → books balance
  - Z.F. damping emerges as critical; MNR '97

Feedback Loops

- Closing the loop of shearing and Reynolds work
- Spectral 'Predator-Prey' Model



Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

→ Self-regulating system → "ecology"

→ Mixing scale regulated

Feedback Loops II

- Recovering the 'dual cascade':
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high k_r ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \left\langle V_{E,\theta}^2 \right\rangle$
    - ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work
    - ⇒ Analogous → inverse energy cascade

- Mean Field Predator-Prey Model

  (P.D. et. al. '94, DI[2]H '05)

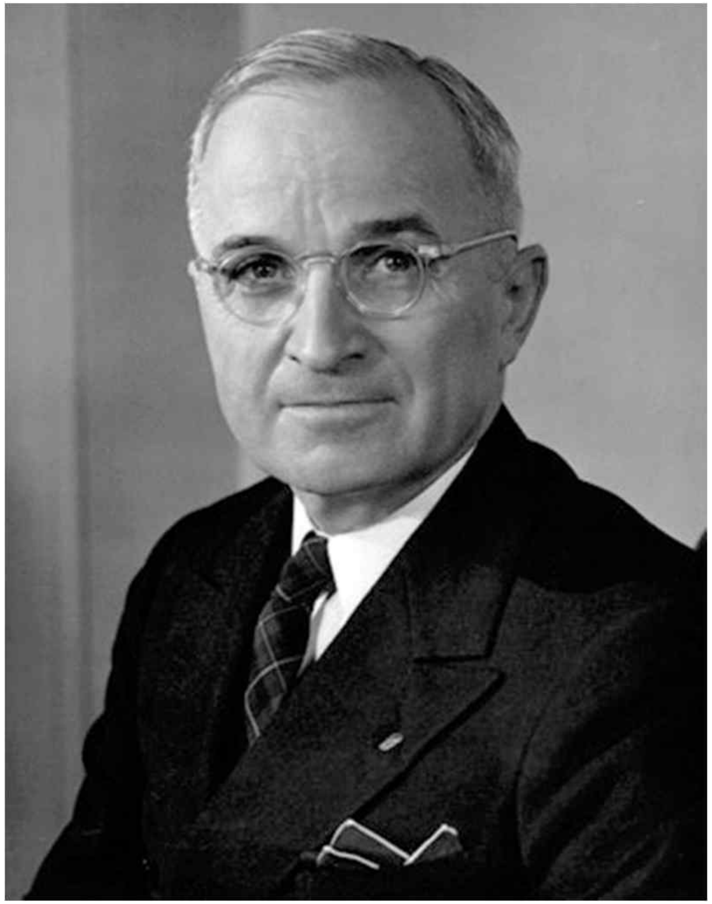
$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_d V^2 - \gamma_{NL}(V^2) V^2$$

System Status

State	No flow	Flow ($\alpha_2 = 0$)	Flow ($\alpha_2 \neq 0$)
N (drift wave turbulence level)	$\frac{\gamma}{\Delta\omega}$	$\frac{\gamma_d}{\alpha}$	$\frac{\gamma_d + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
V^2 (mean square flow)	0	$\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_d}{\alpha^2}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
Drive/excitation mechanism	Linear growth	Linear growth	Linear growth Nonlinear damping of flow
Regulation/inhibition mechanism	Self-interaction of turbulence	Random shearing, self-interaction	Random shearing, self-interaction
Branching ratio $\frac{V^2}{N}$	0	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$
Threshold (without noise)	$\gamma > 0$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$

The Crux of the Matter, …



IV) Pattern Competition!

- Two secondary structures at work:
  - Zonal flow → quasi-coherent, regulates transport via shearing, self-generated, limits scale
  - Avalanche → stochastic, induces extended transport events, enhances scale
- Both flux driven… by relaxation $\nabla T, \nabla n, etc$
- Nature of co-existence?? – who wins?

IV) Staircases

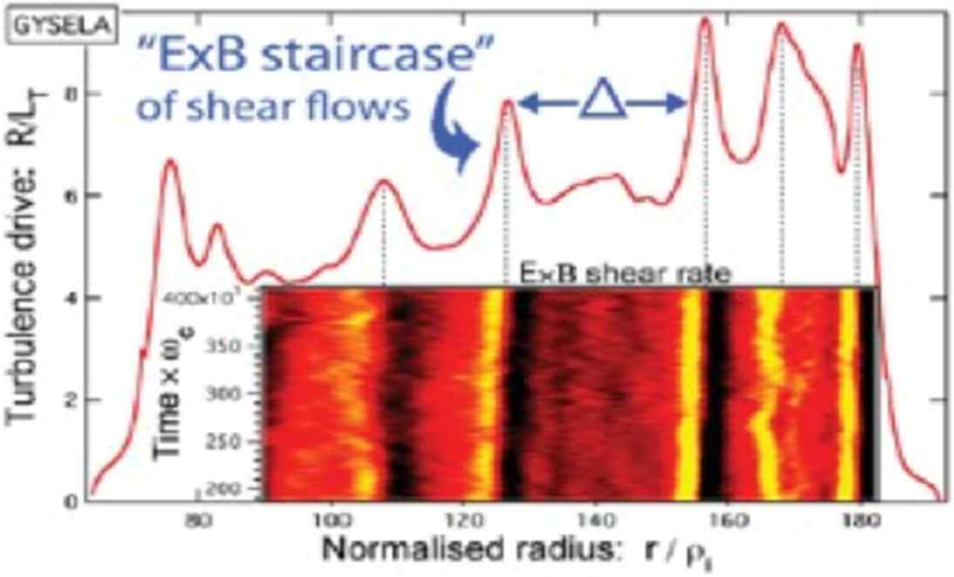
Single Layer →

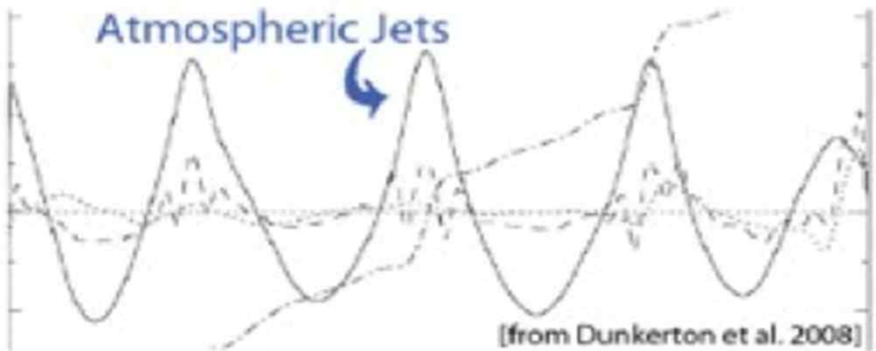
Lattice of Layers + Avalanches

Motivation: ExB staircase formation

- ExB flows often observed to self-organize in magnetized plasmas
- \`ExB staircase’ is observed to form

(G. Dif-Pradalier, P.D. et al. Phys. Rev. E. ’10)





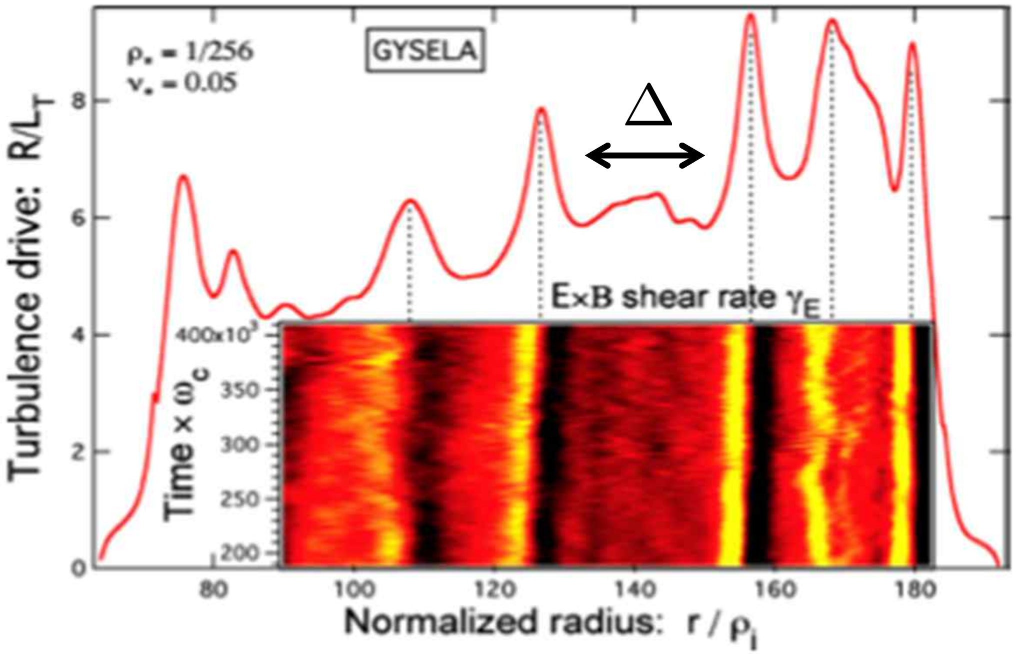
- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases

- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche distribution outer-scale

ExB Staircase

- Important feature: co-existence of shear flows and avalanches



- Seem mutually exclusive ?

  → strong ExB shear prohibits transport

  → avalanches smooth out corrugations

- Can co-exist by separating regions into:

  1. avalanches of the size $\Delta \gg \Delta_c$

  2. localized strong corrugations + jets

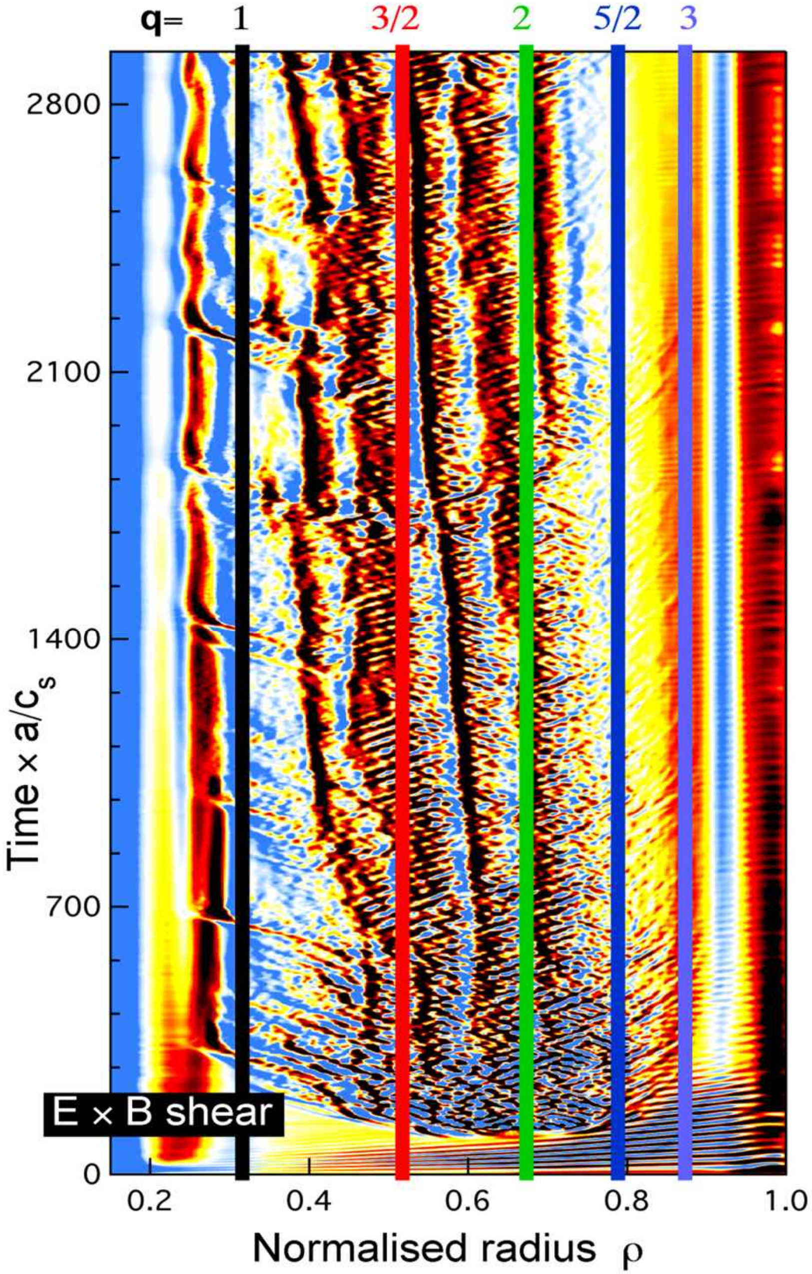
- How understand the formation of ExB staircase??
  - What is process of self-organization linking avalanche scale to ExB step scale?

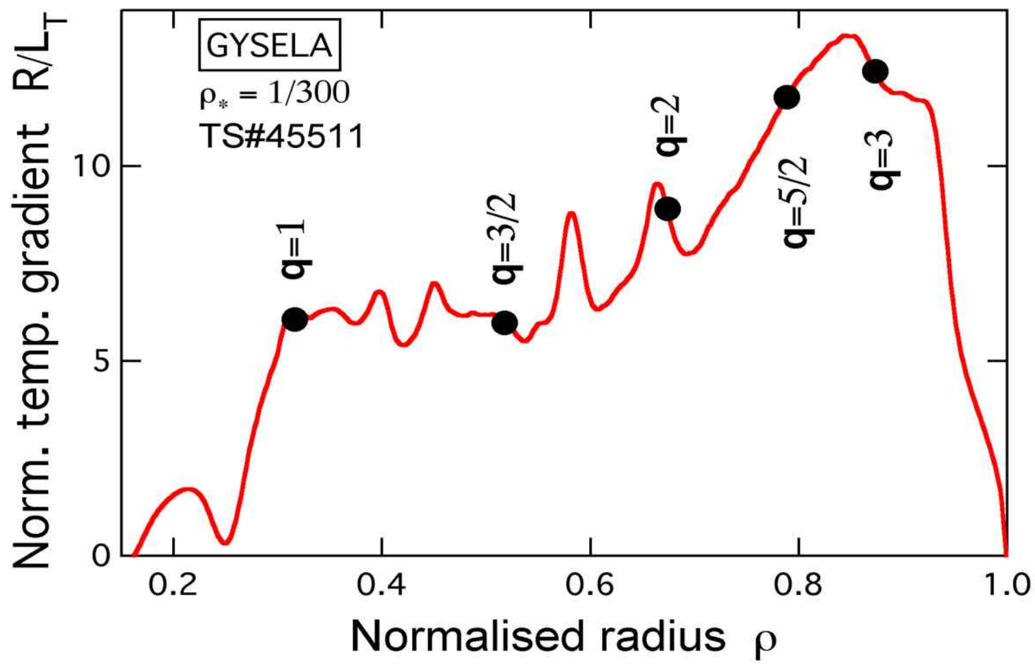
    i.e. how explain the emergence of the step scale ?

- Some similarity to phase ordering in fluids

Corrugation points and rational surfaces

- No apparent relation





➔ Step location not tied to magnetic geometry structure in a simple way

(GYSELA Simulation)

→ Are they real?

E × B staircase visible on fluctuation correlations

Direct exp. characterisation difficult:

flows, profiles & gradients

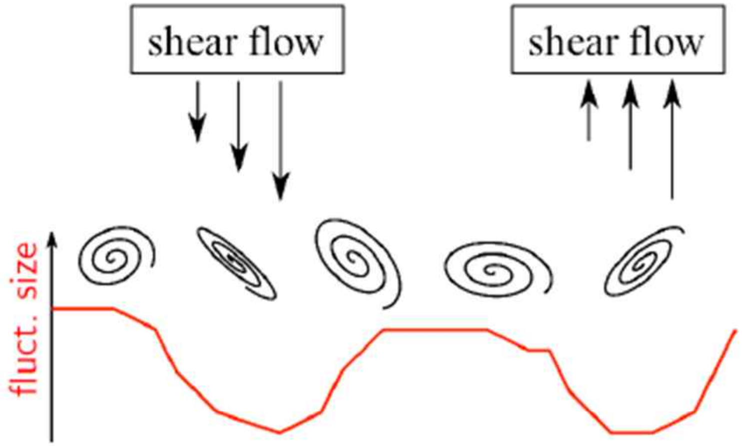
Shear layers in staircase:

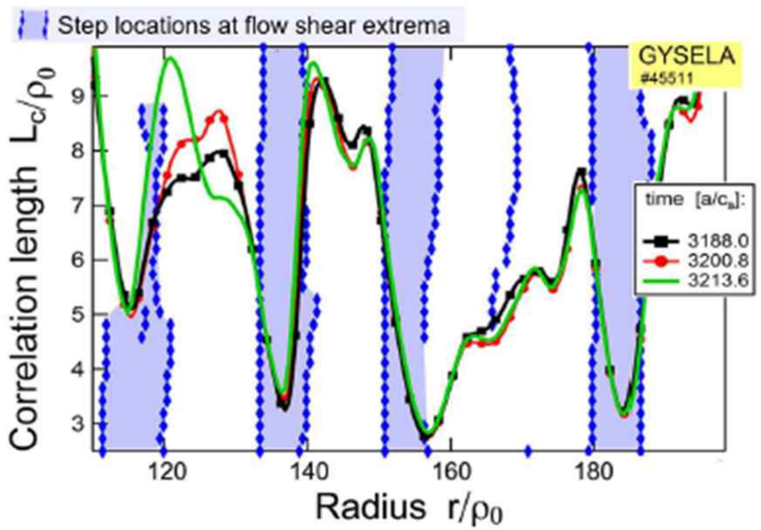
- eddies stretched, tilted, fragmented
- predict quasi-periodic decorrelation turbulent fluct.

$$C_\phi(r,\theta,t,\delta r) = \frac{\langle \tilde{\phi}(r,\theta,t)\,\tilde{\phi}(r+\delta r,\theta,t)\rangle_\tau}{\left[\langle \tilde{\phi}(r,\theta,t)^2\rangle_\tau \,\langle \tilde{\phi}(r+\delta r,\theta,t)^2\rangle_\tau\right]^{1/2}}$$

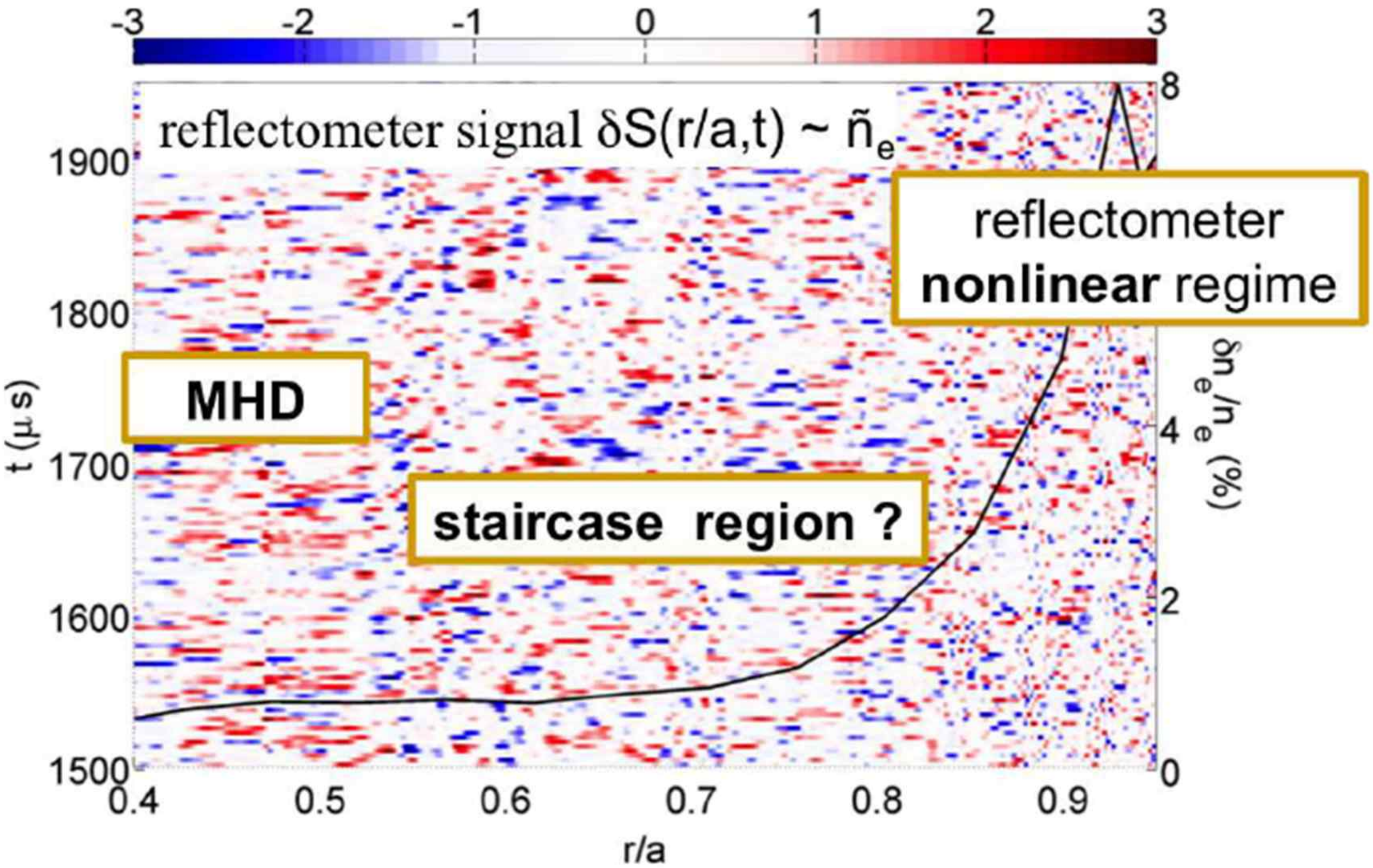
➥ $C_\phi = 1/2$ when $\delta r = L_c$

➥ testable with fast-sweeping reflectometry





Moderate fluctuation level & MHD-free plasmas: optimal for staircase observation

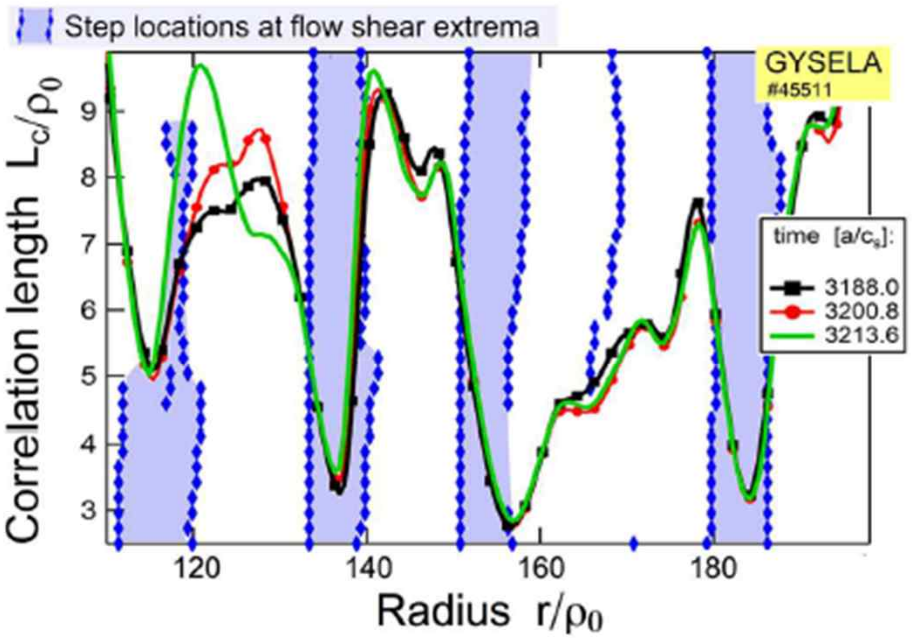


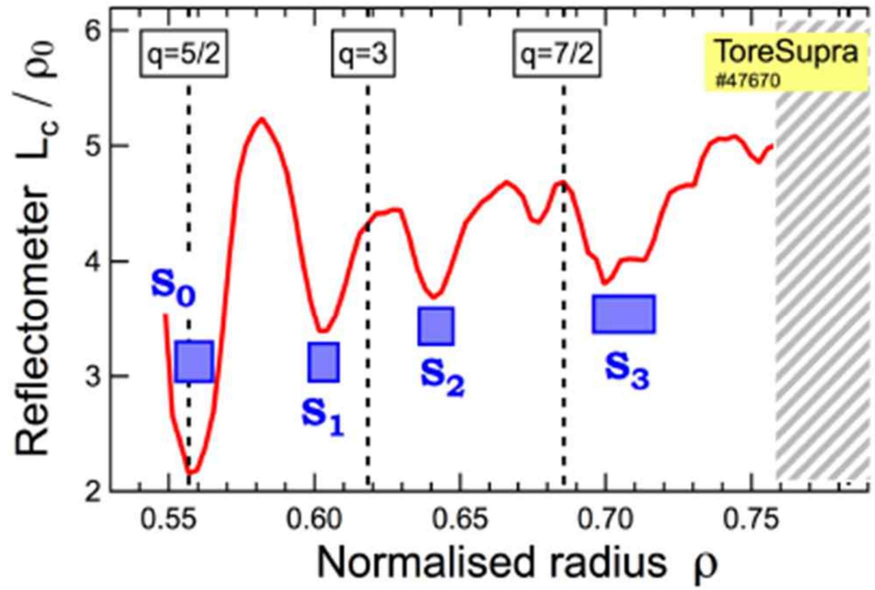
fast-sweeping reflectometry on **Tore Supra** [Clairet RSI 10, Hornung PPCF 13]

➡ localised measure, fast ($\sim \mu s$), sweeping in X–mode : full radial profile δn

➡ **routinely estimate** L_c

Staircase predicted. . .then observed experimentally





- Large set: 179 staircase steps, so far [Dif-Pradalier PRL 15]
  - quasi-regularly spaced radial local minima of L_c
  - reproducible: not random & robust w.r.t. definition of L_c
  - tilt consistent with flow shear around minima
  - no correlation to local q rationals ➠ rules MHD out
  - consistent width [$\sim 10\rho_i$] & spacing [meso.] of local L_c minima

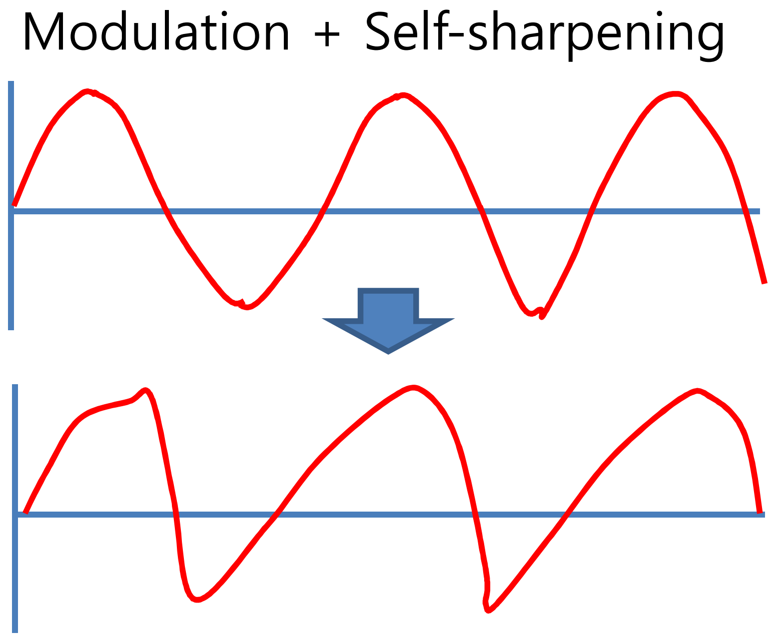
- How to understand it?
  - – Topic for a (theoretical) seminar…
  - – Bi-stable Modulations:
  - – <u>Inhomogeneous mixing</u>

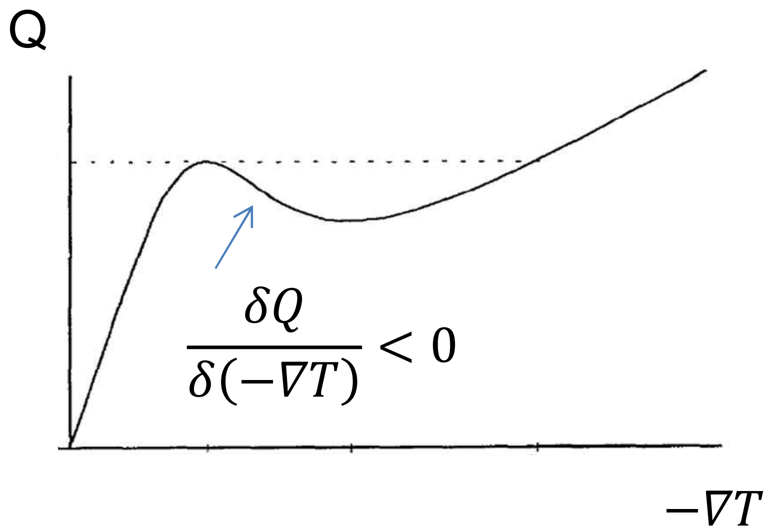
key

➔ "negative diffusion/viscosity"

c.f. also Cahn-Hilliard equation

How?:

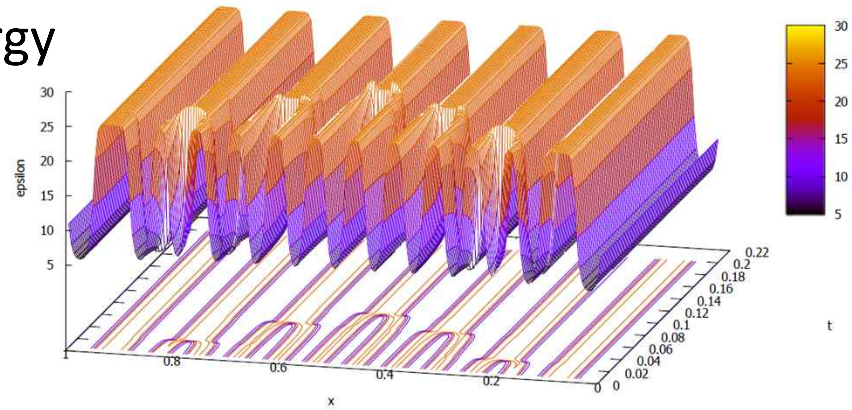




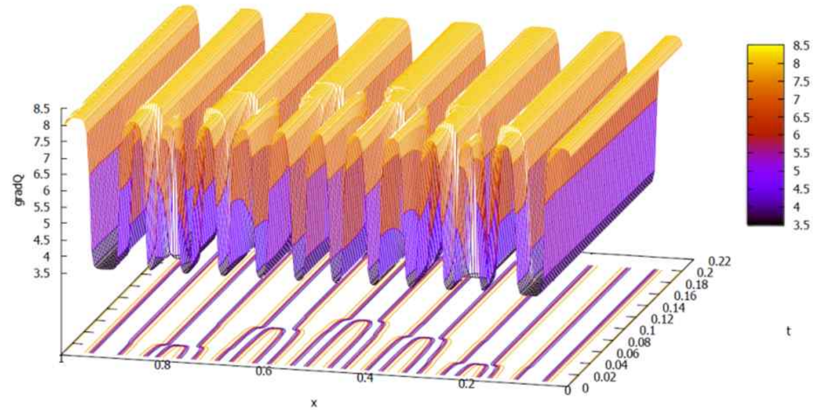
- Bistable flux → l_{mix} (Ashourvan, P.D., 2016-PRE,PoP)
- Jams, ala' traffic flow (Kosuga, P.D., Gurcan – PRL2012)

Staircase Model – Formation and Merger (QG-HM)

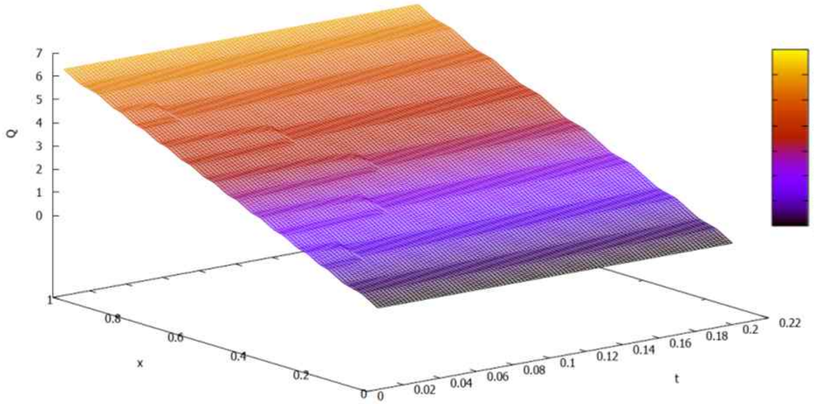
Energy

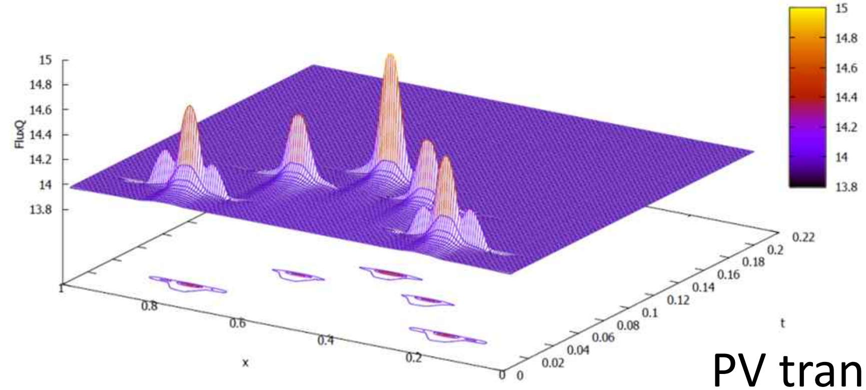


∇q



q





PV transport

- ϵ
- Q_y

} top

- Q
- Γ_q

} bottom

Note later staircase mergers induce strong flux episodes!

$\leftrightarrow$ Avalanching connection?!

Staircase are Dynamic

○Shear pattern detaches and delocalizes from its initial position of formation.

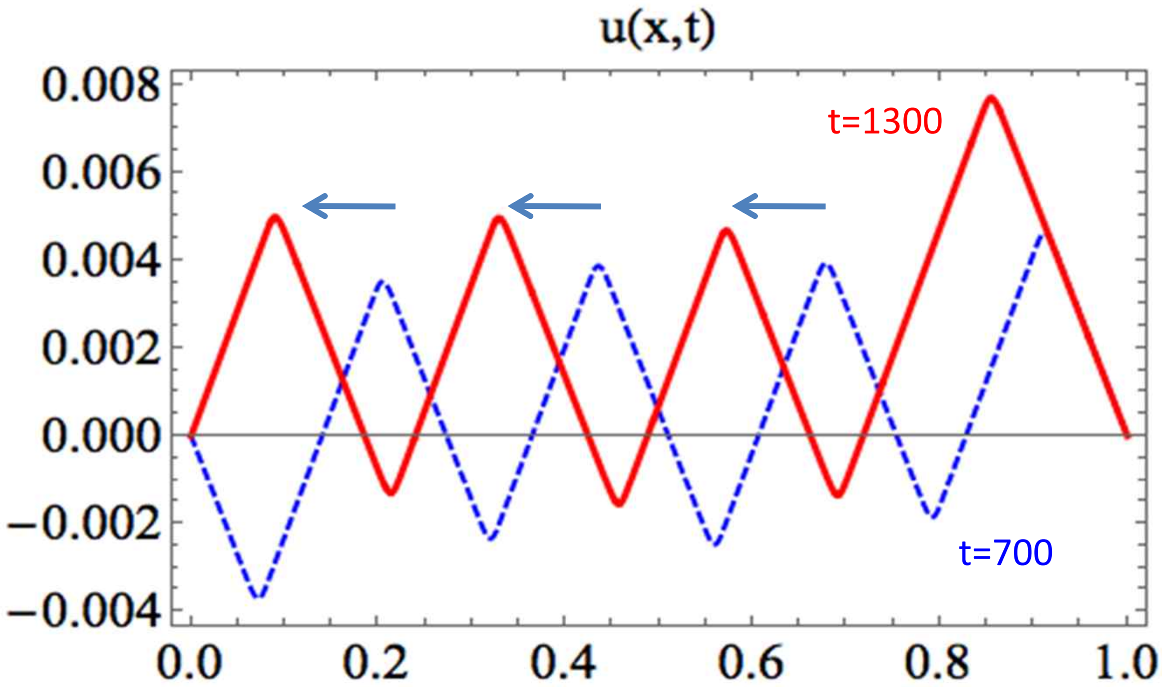
○Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

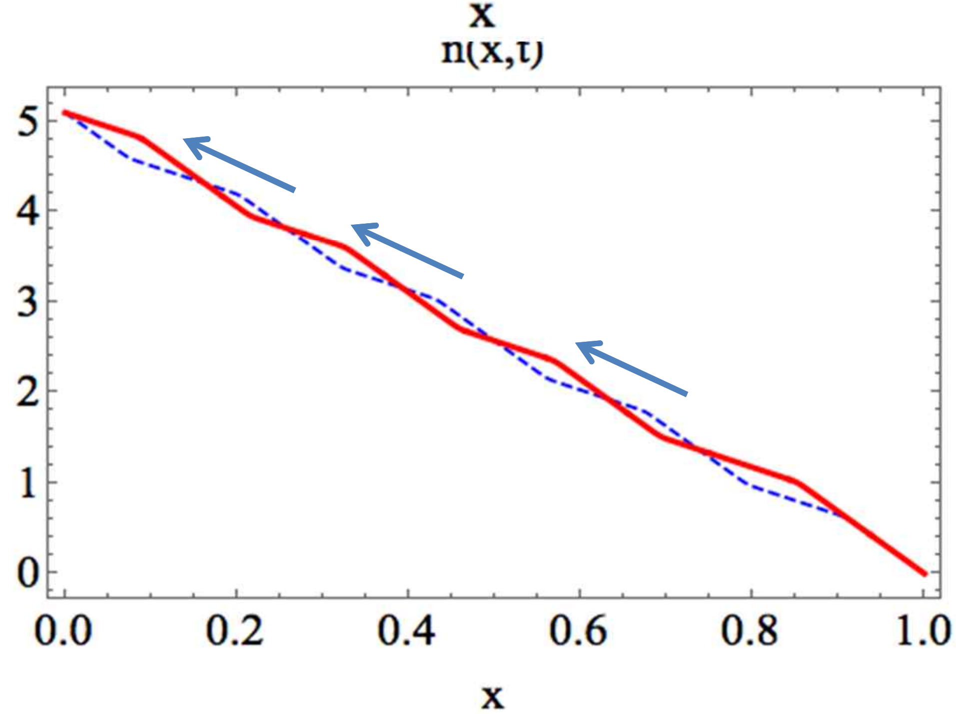
○Shear lattice propagation takes place over much longer times. From t~O(10) to t~(10^4).

○Barriers in density profile move upward in an "<u>Escalator-like</u>" motion.

➔ Macroscopic Profile Re-structuring

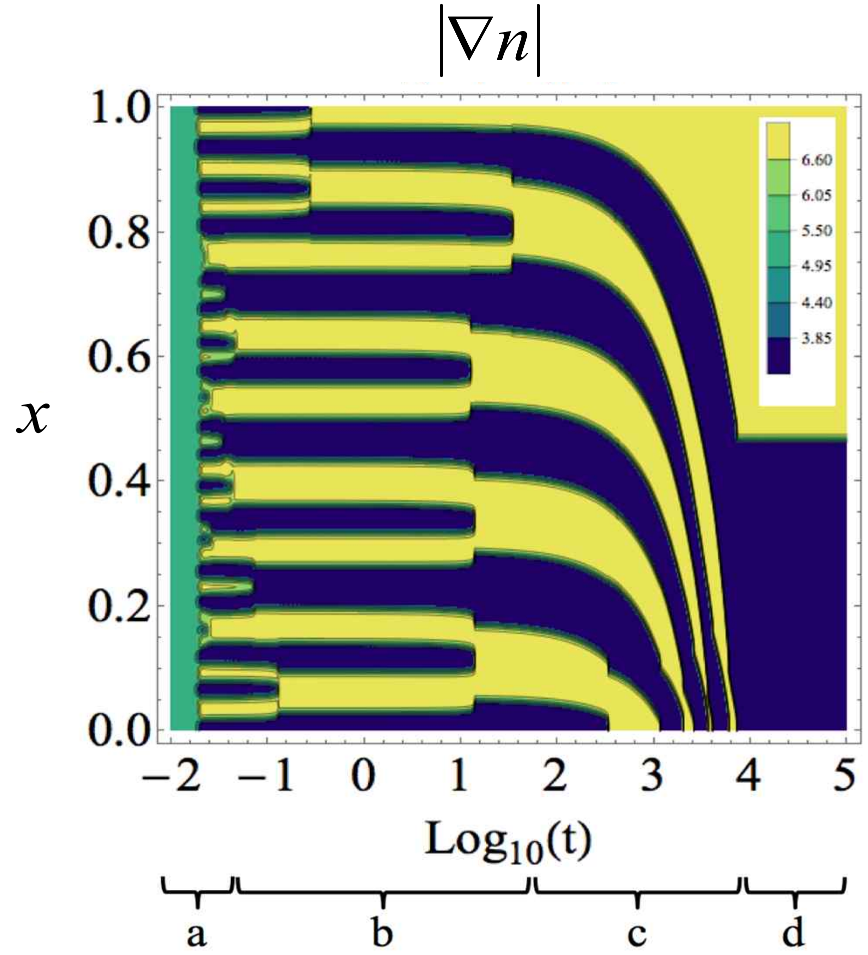
'Non-locality'

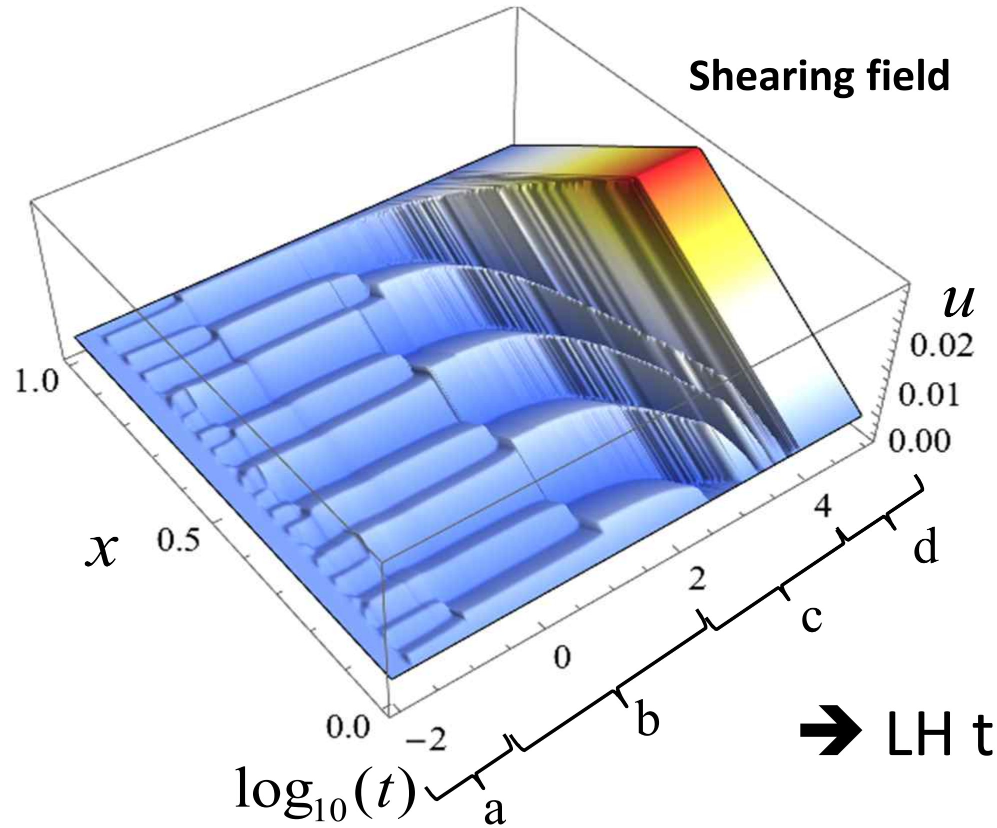




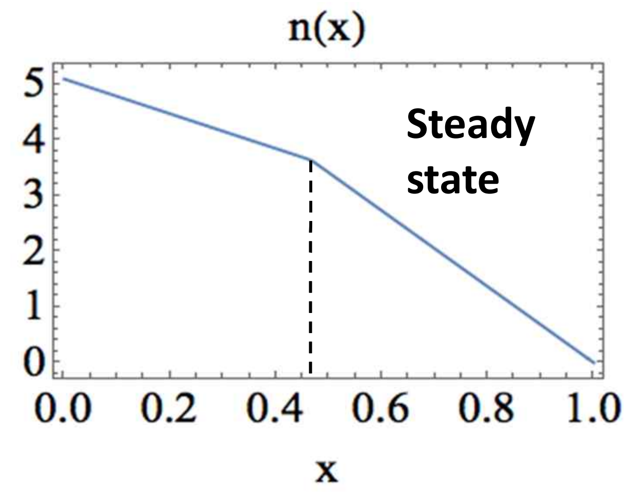
Macro-Barriers via Condensation

(a) Fast merger of micro-scale SC. Formation of meso-SC.
(b) Meso-SC coalesce to barriers
(c) Barriers propagate along gradient, condense at boundaries
(d) Macro-scale stationary profile





➔ LH transition?



Conclusion, of sorts

- Scale selection problem in confined, magnetized plasmas is intrinsically a pattern competition
- Staircase:
  - Naturally reconciles avalanche and shear layers
  - Allows ‘predator and prey’ co-existence via spatial decomposition to separate domains
  - Realizes ‘non-local’ dynamics in transport

Conclusion, of sorts

- Where is confinement physics going?
  - Considerable success in understanding and predicting transport, including bifurcations
  - Evolving:
    - Confinement → Power Handling
    - Transport Reduction → Transport control
  - Need address interaction of turbulence + macro-stability, turbulence with PMI

➔ Boundary optimization, now the central problem

- This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Science, under Award Number DE-FG02-04ER54738