Physics 235

→ Notes 3: Transport Channels, Particle Transport, Ambipolarity

What about particle, momentum and ion thermal transport in Stochastic Magnetic Fields?

→ Relevant to RMP, L→H transition, etc.

→ Channels { heat, particles, momentum → e, i

– Particles { why speak of heat transport? test particle diffusivity calculated. Why not particle transport?

Note: The 'point' of stochastic fields is

$\chi_\perp \sim v_{the}\, D_M$

↳ electron speed.

Useful to:

→ ↗ consider kinetic eqn.

↘ distinguish between:

a) self-consistent case

$\tilde{B}$ produced by $\tilde{J}_\parallel$ in plasma

(i.e. EM instabilities – Ampere's Law)

b) $\tilde{B}$ produced by external

means (coil) – i.e. RMP, though plasma response (i.e. screening) significant

Now,

$$\frac{\partial f}{\partial t} + v_\parallel \nabla_{\parallel 0} f + v_\parallel \frac{\delta B}{B_0} \cdot \nabla_\perp f + \cdots = 0$$

then integrating;

$$\frac{\partial \langle n_e \rangle}{\partial t} + \nabla_\perp \cdot \left\langle \frac{\delta B_r}{B_0} \frac{\tilde{J}_{\parallel e}}{(-n_0 |e|)} \right\rangle + \cdots = 0$$

but

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} \tilde{J}_\parallel$$

$$= -\frac{4\pi}{c} \left(\tilde{J}_{\parallel e} + \tilde{J}_{\parallel i} \right)$$

$$\tilde{J}_{\parallel e} = -\frac{c}{4\pi} \nabla_\perp^2 A - \tilde{J}_{\parallel i}$$

$$= -\frac{c}{4\pi} \nabla_\perp^2 A_\parallel - n_0 |e| \hat{V}_{\parallel i}$$

$\Uparrow$
ion flow

so/

$$\frac{\partial \langle n_e \rangle}{\partial t} + \nabla_\perp \cdot \left\langle \frac{\delta B_r}{B_0} \left(-\frac{c}{4\pi} \nabla_\perp^2 \hat{A}_\parallel - n_0 |e| \hat{V}_{\parallel i} \right) \right\rangle + \cdots$$
$$= 0$$

$$\delta B_r = \nabla_\theta \hat{A}_\parallel$$

$\Rightarrow$

$$\frac{\partial \langle n_e\rangle}{\partial t} + \frac{\partial}{\partial r}\left\langle \nabla_\theta A_\parallel \left(-\frac{c}{4\pi}\nabla_\perp^2 \hat{A}_\parallel - n_0 e d\, v_{\parallel i}\right)\right\rangle$$

$+ \qquad = 0$

so

$$\frac{\partial \langle n_e\rangle}{\partial t} = \frac{c}{4\pi}\frac{\partial}{\partial r}\overset{①}{\left\langle (\nabla_\theta \hat{A}_\parallel)(\nabla_\perp^2 \hat{A}_\parallel)\right\rangle}$$

$$+\, n_0 e d\, \overset{②}{\left\langle \delta B_r\, \hat{v}_{\parallel i}\right\rangle}$$

① : Taylor identity calculation (akin $\langle v_r \nabla^2 \phi\rangle$)

$$\langle (\nabla_\theta \hat{A}_\parallel)(\partial_r^2 A_\parallel + \underbrace{\cancel{\nabla_\theta^2 \hat{A}_\parallel}}_{\text{odd}}) \rangle$$

$$= \langle (\nabla_\theta A_\parallel)(\partial_r^2 A_\parallel)\rangle$$

$$= \langle \partial_r (\nabla_\theta A_\parallel)(\partial_r A_\parallel)\rangle - \overset{\text{odd}}{\cancel{\langle (\nabla_\theta \partial_r A_\parallel)(\partial_r A_\parallel)\rangle}}$$

$$= \partial_r \langle (\nabla_\theta \hat{A}_\parallel)(\partial_r A_\parallel)\rangle$$

$$= -\partial_r \langle (\delta B_r)(\delta B_\theta)\rangle$$ [Magnetic stress.

phase $\langle k_r k_\theta \rangle$ crucial

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$$\frac{\partial \langle n_e \rangle}{\partial t} = -\frac{c}{4\pi} \partial_r^2 \langle (\delta B_r)(\delta B_\theta) \rangle$$

→ magnetic stress

$$+ n_0 \partial_r \langle \delta B_r \hat{V}_{\parallel i} \rangle$$

→ magnetic "flutter" of parallel ion flow.

Points:

— no explicit dependence on small electron inertia ; i.e. $1/m_e$.

— relation of magnetic stress to electron particle transport due stochastic fields

i.e. [suggests effect on zonal flows, as zonal flows are fundamentally charge transport effect.

— fluctuating parallel ion flows + magnetic tilt ⇒ change electron density. (see Chernikov) Finn, Guzdar

also Chen, PD, Tobias.

N.B. For external field, key is to calculate:

- δB_r in plasma
- $\tilde{J}_{\parallel 0}$ in plasma.

δB_r in $\sim \dfrac{\delta B_{ext}}{G}$

→ Re: zonal flows:

Point: Magnetic wandering impacts zonal flows via charge

see Chen, PD, Tobias ApJ. 2020

Recall: - fluctuations → DW model i.e. H-M, etc.

- zonal mode: charge separation, radially - ambipolarity breaking

$\underline{\nabla} \cdot \underline{J} = 0$

usual e.s.; $\nabla_r J_{r\,pol} = 0$

⇒

$$\frac{\partial}{\partial t} \nabla_\perp^2 \phi + \underline{V} \cdot \underline{\nabla}\, \nabla^2 \phi = 0$$

linear pol. drift

NL pol. drift

so, for $\langle \ \rangle$ → zonal symmetry

[redacted] $\left[\int \text{ion gk} - \int \text{electron gk} \right]$

$$\frac{\partial}{\partial t}\langle \nabla_r^2 \phi \rangle + \partial_r \langle \tilde{V}_r \nabla_r^2 \tilde{\phi} \rangle + \dots = 0$$

↓ polarization charge Q_{pol}.

↓
- flux of polarization charge (i.e. net difference ions electrons)
- relate to Reynolds stress by Taylor identity
- compute via modulation

so, can view:

$$\partial_t \langle Q_{pol} \rangle + \partial_r \langle \tilde{V}_r Q_{pol} \rangle = 0$$

→ $$\Gamma_{charge} = \left(\langle \tilde{v}_r \tilde{n}_{i,GC} \rangle + \langle \tilde{v}_r \tilde{Q}_{pol} \rangle \right) |e| - \left(\langle \tilde{v}_r \tilde{n}_{e,GC} \rangle \right) |e|$$

$$\hat{n}_i = \hat{n}_e \Rightarrow \Gamma_{charge} = 0 \quad \checkmark .$$

$n_{i,gc} + n_{i,pol}$

But magnetic perturbations ~~[illegible]~~ induce new mean charge transport!:

$$\Rightarrow \underline{\nabla} \cdot \underline{J} = 0$$

$$\underline{\nabla_\perp} \cdot \underline{J}_{pol} + \underline{\nabla_\parallel J_\parallel} = 0$$

$$\nabla_\parallel = \nabla_\parallel^{(0)} + \frac{\delta \underline{B}}{B_0} \cdot \nabla_\perp$$

$$\underbrace{\frac{\partial}{\partial t} \langle \nabla^2 \phi \rangle + \langle \underline{\tilde{v}} \cdot \nabla \nabla^2 \tilde{\phi} \rangle}_{} = \langle \underline{\tilde{B}} \cdot \nabla \tilde{J}_\parallel \rangle$$

$\partial_r \langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle$ $\quad$ $\partial_r \langle \tilde{B}_r \tilde{J}_\parallel \rangle$

obvious from 2D MHD

ion pol. charge transport $\Rightarrow$ advection

current flow along tilted lines

(n.b. current: electrons $\neq$ ions)

so

$$\partial_t \langle \nabla^2 \phi \rangle = -\partial_r \left\{ \langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle - \langle \tilde{B}_r \tilde{J}_\parallel \rangle \right\}$$

so current flow along tilted lines acts to affect charge balance

$\Rightarrow$ enters effect in Z.F.

Note:

- sign not clear apriori
- for Alfvenic fluctuations $\Rightarrow$ damps
  i.e. acts cancel e.s.
- obviously related to electron particle transport.

Other:

→ electron momentum transport

→ "electron viscosity" / "hyper-resistivity"

i.e.
$$E_{\parallel} = \eta J_{\parallel} - \nabla_{\perp} \cdot \mu_e \cdot \nabla_{\perp} J_{\parallel}$$

$\downarrow$ (?!)

$$\frac{c^2}{\omega_{pe}^2} v_{Th} \Delta_M .$$

⇒ Reconnection rate

i.e. Sweet-Parker:

$$V \sim V_A \frac{\Delta}{L}$$

$$\sim 1/S^{1/2} \, V_A$$

$$= V_A / \sqrt{R_m}$$

Electron viscosity:

$$V \sim V_A / (R_{m\mu})^{1/4}$$

→ Physics of Hyper-resistivity is essential to ELM crash.

→ One take: Drake '95

ITG : PSFI :: ETG : ∇J-driven

$\mu = \mu(\nabla J)$ → nonlinearity!

⇒ Good topic for further research.