Physics 239

Lecture 17 – Violent Relaxation

(c.f. read posted paper by D. Lynden-Bell)

Recall:

① → Galaxy = Large N ensemble of stars (+ more, in reality)

② → Model of Galactic Dynamics ⇒

(Collisionless) Vlasov – Poisson System

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla} f - \underline{\nabla}\phi \cdot \frac{\partial f}{\partial \underline{v}} = 0$$

$$\nabla^2 \phi = 4\pi G \int f \, d^3v$$

[Continuity in Phase Space for Hamiltonian Trajectories]

③ → Stationary Solution:

$$f = f(J_1, J_2, J_3)$$

(excluding resonances)

Strong Jeans Thm.

⇒

Integro-differential Poisson Eqn. :

$$\nabla^2 \phi = 4\pi G \int d^3v \, f(\phi \mid \underline{J})$$

→ general

ansatz's : Spheroidal Distribution
King Model
Isothermal Sphere
⋮

[BGK mode problem.

n.b. how relevant ? - i.e. Gaussian ?

→ Clearly limited.....

and :

(4) → Jeans Equations - not closed
(Hydro) (Equation of state ?)

- useful for macroscopics.

i.e. Oort Limit.
Vlasov → Fluid → #'s

(5) → Virial Theorem : Basic macroscopic constraints.

⑥ → Novelty: Collisionless Damping of Waves / Modes
→ Landau Damping

i.e. $\epsilon = 1 + \frac{\omega_p^2}{k}\int dv \, \frac{\partial f_0/\partial v}{\omega - kv}$ plasma (generalizes $\epsilon = 1 - \frac{\omega_p^2}{\omega^2}$)

collective response

$\epsilon = 1 - \frac{4\pi G}{k^2}\int d^3v \, \frac{\underline{k}\cdot\partial f_0/\partial \underline{v}}{\omega - \underline{k}\cdot\underline{v}}$ {gravitating matter

Difference: - attraction, repulsion
- $f_0 \to f_{Max}$ ..
damping → slope.

⇒ $\epsilon_{Im} \leftrightarrow \delta(\omega - \underline{k}\cdot\underline{V})$ ⇒ wave-particle resonance.

dissipation

→ Physics: Wave power dissipated in resonant particles by:

$\langle E\cdot J\rangle$ or $\langle \underline{F}_g\cdot\underline{V}\rangle$

$-\nabla\phi$ $\quad$ $\nabla\phi_g$.

→ see also: $\left\{ \begin{array}{l} \text{2/8a notes} \\ \text{Fall 2018.} \end{array} \right.$

(7)

→ What happens to the energy ?

→ heats distribution

→ if evolve $\langle f \rangle$ → coarse grained distribution

$$\partial_t \langle f \rangle = \partial_v D \partial_v \langle f \rangle$$

"quasilinear equation"

$$D = \sum_k \frac{q^2}{m^2} |E_k|^2 \left\{ \pi \delta(\omega - kv) + \frac{|\gamma_k|}{(\omega_k - kv)^2} \right\}$$

from: $\langle E \delta f \rangle = \langle E \, \delta f_{\substack{\text{Lin} \\ \text{Resp}}} \rangle$

requires multiple modes ⇔ resonance overlap.

See $\left\langle \begin{array}{l} \text{P.D. Itoh + Itoh} \\ \text{Chapter 3.} \end{array} \right.$

All of this poses many questions:

- How understand relaxation in collisionless system?

N.B. Some evidence that relaxation happens

i.e. Light distribution in elliptical galaxies is quite regular

⇒ suggests galaxies reached some sort of "equilibrium" – i.e. attracting state.

but how / why?

⇒ collisional relaxation way too slow

* ⇒ how reach an 'equilibrium'? Is it an 'equilibrium'? Characterize.

- Equivalent: if fed with:

$$\nabla^2\phi = 4\pi G \int d^3v\, f(\phi|\underline{J})$$

Zoology of models ⇒ many solutions.

So issue is: - **What is most probable solution?** *

- Need assess likelihood

⇒ entropy

→ Specific Questions:

① - entropy - how define? — Partition ⇔ coarse graining (count states)

- Phase space density conservation ⇒ exclusion! - similar to Fermi-Dirac statistics (but classical!).

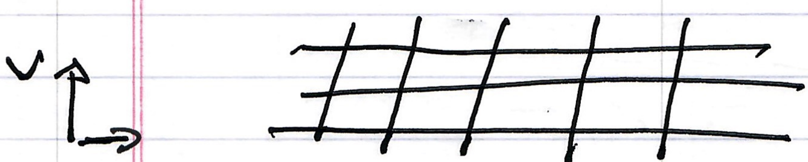
② Relaxation → how??

Routes to collisionless relaxation:

ⓐ

→ Coarse graining + chaos

- Coarse graining → partition on phase space



→ cell

- Chaos → exponentially diverging trajectories
(in some sense unstable to initial conditions)

$$\sigma_{KS} = \sum_{\substack{i \\ h_i > 0}} h_i$$ → positive Lyapunov exponent(s)

i.e.

"active" cells increases in time (exponentially)

$\Rightarrow$

δ(Information) > 0 $\Rightarrow$ entropy produced as information produced.

N.B.: Chaos - Entropy Production tied to phase space partition.

Also observe: if Vlasov

- exact phase space density conserved

i.e. $\frac{df}{dt} = 0$, phase volume conserved

- then, as phase volume increases during evolution coarse grained distribution must decrease

i.e. const $\sim f A_0 = A \langle f \rangle$

so

$$\frac{\partial \langle f \rangle}{\partial t} < 0$$

i.e. Consider quasilinear equation

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} D \frac{\partial \langle f \rangle}{\partial v}$$

so

$$\frac{\partial \int \langle f \rangle^2}{\partial t} = -\int D \left(\frac{\partial \langle f \rangle}{\partial v} \right)^2 \leq 0$$

decreases. ⇒ consequence of coarse graining

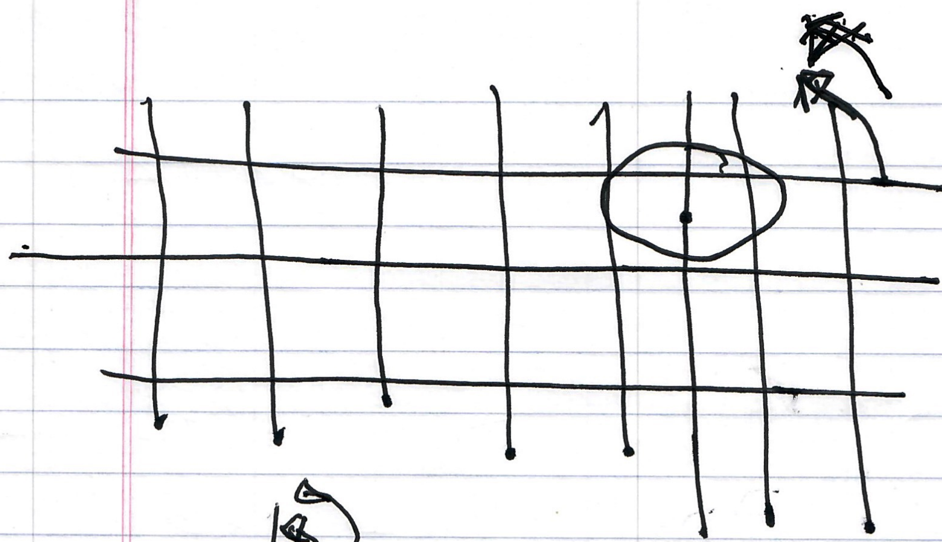
(h) "Phase Mixing" and Coarse graining

⇒ What is 'phase mixing'?

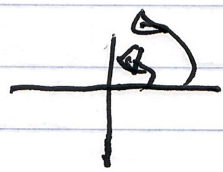
- phase element stretching and winding

+

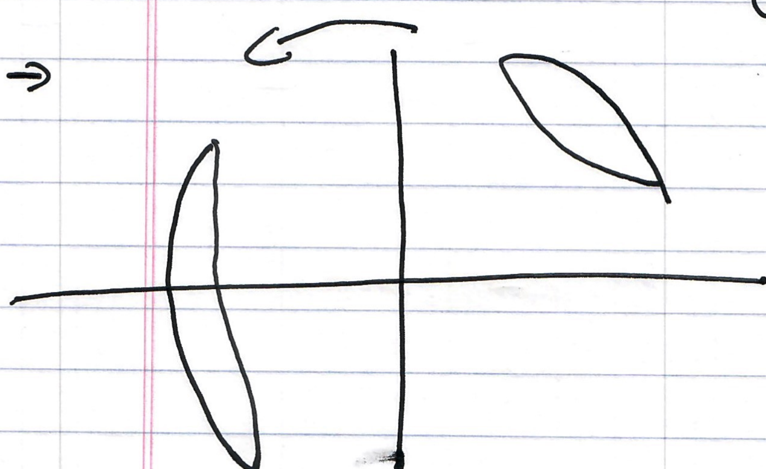
- coarse graining



differential rotation!



eddy in shearing flow (akin homogenization)



rule: if any part cell occupied → cell is occupied.

i.e. system is shearing flow + partition → equivalent to flow + diffusion

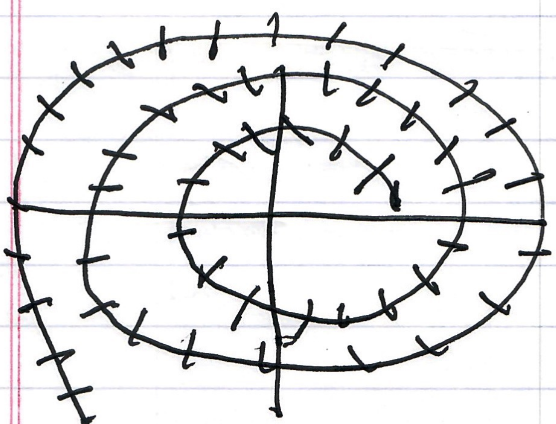
i.e.
$$\frac{\partial \omega}{\partial t} + \underline{v} \cdot \underline{\nabla} \omega - \nu \nabla^2 \omega = 0$$

$$\underline{v} = \underline{\nabla} \phi \times \hat{z}$$
$$\omega = \nabla^2 \phi$$

then, after several windings:

"wound noodle of finite thickness"

i.e



i.e filament wound many times

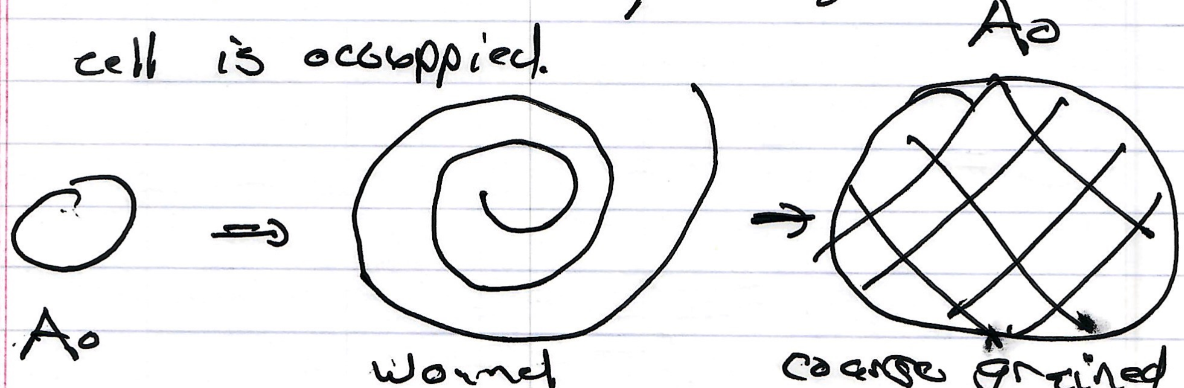
→ wound thick noodle →

thickness begins hit partition scale

→ partition cell is minimum resolution ⇒

if flow hits any cell, that cell is occupied.

i.e



obviously:

- exact area conserved

but

- $A_0 \ll \langle A \rangle$, as coarse grained area increases, for long time

- but $f A_0 = \langle f \rangle \langle A \rangle$ $\Rightarrow$ phase space density conservation

so

$\langle f \rangle = \frac{A_0}{\langle A \rangle} f$ $\Rightarrow$

$\langle f \rangle < f \Rightarrow$ decay, or relaxation of $\langle f \rangle$.

$\Leftrightarrow$ 'phase mixing' with coarse graining

$\Rightarrow$ collisionless relaxation