Physics 239

Lecture 12 - Planetesimals 1

→ Planet formation:

1 μ dust particle → Planet

⇝ 10^{12} factor in size

→ Planetesimal: Bodies 1-10 km scale

→ Sequence:
- Planetesimal
↓
- Terrestrial Planet
↓
- Giant planet

- Here focus on planetesimal formation

i.e. $\boxed{1\,\mu \rightarrow 1\text{ km.}}$? How? Time scale? What happens?

- Closely linked to disk physics. → turbulence

Later evolution: Planets class.

→ Drag on small particle

Scales for neutral gas: (collisional regime)

$$\underset{\substack{\downarrow \\ \text{range} \\ \text{intermolecular} \\ \text{force.}}}{r_{force}} \underset{\text{dilute}}{<} \bar{n}^{-1/3} < \underset{\substack{\downarrow \text{ gas} \\ 1/n\sigma}}{l_{mfp}} < L$$

Plasma:

$$\bar{n}^{-1/3} < \underset{\substack{\downarrow \\ \text{Debye Length}}}{\lambda_{De}} < l_{mfp} < L$$

Then: for neutral gas:

– $l_{grain} < \underset{gas}{l_{mfp}}$ ⇒ grain sees ensemble of individual particles

⇒ grain feels Epstein drag

→ particle collisions

[redacted]

— $l_{mfp} < l_{grain}$ ⇒ grain (feels / sees) 'fluid' of gas particles

tiny ↓

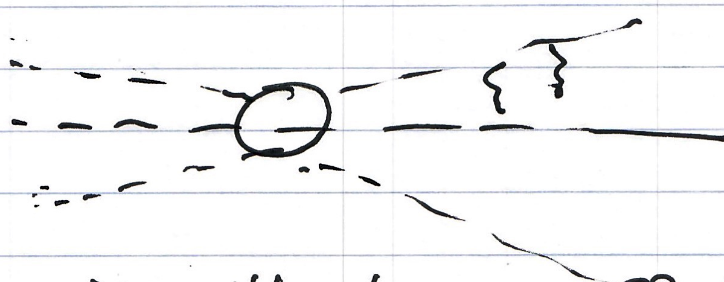
⇒ grain feels Stokes Drag.

for $Re_{grain} \sim \frac{\tilde{v}\, l_{grain}}{\nu} \ll 1$

$\tilde{v}$ is turbulent v

viscous fluid drag.

⇒ rare instance of low Reynolds # flow physics in Astrophysics

⇒ 

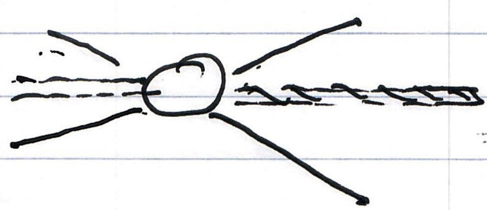
→ grains will settle to disk midplane.
turbulent sedimentation

→ time scale?

⇒ effect of turbulence on settling?

→ what else happens?

- coagulation
- infall
- instability
  - KH
  - self-gravitating sheet.



→ Drag — Aerodynamic, on dust

— Epstein $\qquad l_{dust} < l_{mfp,\,gas}$

gas V_{th}

dust particle

$$V_{th} \sim \left(\frac{8 k_B T}{\pi \mu m_H}\right)^{1/2}$$

μ: mean molec. wt.

— # hit front: $\quad f_+ \sim \pi s^2 (V + V_{th}) \dfrac{\rho}{\mu m_H}$

— # hit behind:

$$f_- \sim \pi s^2 (V_{th} - V) \frac{\rho}{\mu m_H}$$

and momentum transfer per collision

$$\Delta p \sim 2 \mu m_H V_{th}$$

so

$$\underbrace{F_D}_{\text{force drag}} \cong -s^2 \rho V_{th} V \sim \cancel{2\mu m_H} V_{th} \left(\pi s^2 (V + V_{th}) \frac{\rho}{m_{mol}} - \pi s^2 (V_{th} - V) \frac{\rho}{\cancel{m_H}} \right)$$

$$\boxed{\underline{F}_D \cong -\pi \sigma^2 \rho V_{th} \underline{V}}$$

factor ∞ $\frac{4\pi}{3}$

$$\boxed{\eta = \rho \nu}$$

– Stokes

→ Recall: $\underline{F}_D = -6\pi \eta a \underline{V}$

for sphere of radius a in stokesian hydro.

→ more generally: $\underline{F}_D \sim -\eta \ell \underline{V}$

ℓ → largest scale

→ Lengthy, but exact, calculation (see Landau & Lifshitz Fluids)

→ Can approach by (~) dimensional analysis,

$$F_D \simeq C_D \overbrace{\frac{\pi s^2}{2}}^{A_s} \rho V^2$$ opposed motion

C_D → Drag coefficient

so

$$\boxed{\underline{F}_D \simeq -C_D \frac{\pi s^2}{2} \rho V \underline{V}}$$

C_D → function dimensionless ratios, only.

Expect, for $Re \sim Vs/\nu < 1$, $F_D \sim V$.

So $C_D \sim 1/Re$ ⇒

$$\underline{F}_D \simeq -\frac{\pi \cancel{s^2}}{2} \frac{\nu}{\cancel{V}\cancel{s}} \rho \cancel{V} \underline{V}$$

$$\simeq -\frac{\pi}{2} \eta \, \underline{V}$$

→ Stokes, up to #.

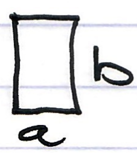
Stokesian Hydro:

$$-\eta \nabla^2 \underline{v} = -\underline{\nabla} p$$

$$\underline{\nabla} \cdot \underline{v} = 0$$

reversible

Exercise:



$b > a$

Show drag same face-on (pressure) and edge-on (viscosity)

$$\underline{F}_D \sim -\eta b \underline{v}$$

→ Settling

Define frictional time scale for particle

$$t_{fric} \sim \frac{m v}{|F_D|} \qquad m = \frac{4\pi}{3} s^3 \rho_m$$

ρ_m → material (i.e. dust)

$$t_{fric} \sim \frac{\frac{4\pi}{3} s^3 \rho_m v}{|-\pi s^2 \rho v_{Th} v|}$$

$$\boxed{t_{fric} \sim s \frac{\rho_m}{\rho V_{th}}}$$

#'s: $r = 1\,AU$
$\rho \sim 10^{-9}\,g/cm^3$
$\rho_m \sim 3\,g/cm^3$
$V_{th} \sim 10^5\,cm/sec$

$\Rightarrow \boxed{t_{fric} \sim 3\,sec}$ $\rightarrow$ tight coupling to flow.

Dust
Particles very tightly coupled to flow.

Now: for settling time:

- expect particle comes to terminal velocity quickly ($t_{fric} \sim 3\,sec$)

- $\underline{F}_D = -\frac{4\pi}{3} \rho s^2 V_{th} \underline{V}$

$$\underline{F}_{grav} = -m\Omega^2 z\,\hat{z}$$

$\therefore$

$$\underset{\frac{4\pi}{3}s^3\rho_m}{m}\,\Omega^2 z \sim \frac{4\pi}{3}\rho s^2 v_{th} v$$

neglects turbulence

$$\boxed{v_{settle} \sim \frac{\rho_m}{\rho}\frac{s}{v_{th}}\Omega^2 z}$$

note larger particles settle faster

$\sim .06$ cm/sec.

$$\boxed{t_{settle} \sim \frac{z}{v_{settle}} \sim \underline{\underline{10^5}}\,yr.}$$

→ expect dust particles to settle out of disk on

$t \sim 10^5$ yr $\ll$ Life disk.

N.B.: $t_{settle} \sim \frac{\cancel{z}\, \rho\, v_{th}}{\rho_m\, s\, \Omega^2 \cancel{z}}$

$$\sim \frac{\rho\, c_s}{\rho_m\, s\, \Omega^2} \qquad v_{th} \sim c_s$$

$$\sim \left(\rho \frac{c_s}{\Omega}\right) \frac{1}{\rho_m s \Omega}$$

$$\boxed{t_{sett} \sim \left(\Sigma / \rho_m s \Omega\right) \exp\left(-z^2/2H^2\right)}$$

$\Rightarrow$ dust particles settle out of the upper disk very rapidly.

$\therefore$ Dust particles settle out of disk on $t < t_{life}$ disk. $\uparrow$ upper Dust on.

parts settle faster out

$\Delta \Rightarrow$ Key: - Drag scalings !

- Neglect turbulence

→ But Particles will coagulate while settling!

a la Smoluchowski

→

larger particles will settle faster than small and sweep up smaller particles en route.

↝ big get bigger.

⇒

$$\frac{dm}{dt} = \underbrace{\pi s^2 |V_{settle}|}_{\text{swept volume}} \; \overbrace{\rho(z)}^{\text{gas density}} \; \underbrace{f}_{\rho_{dust}/\rho_{gas}}$$

$$V_{settle} \simeq \frac{\rho_m}{\rho} \frac{s\Omega^2 z}{v_{th}}$$

so

$$\frac{dm}{dt} = \pi s^2 \frac{\rho_m}{\cancel{\rho}} \frac{s\Omega^2 z}{v_{th}} \cancel{\rho} f$$

$$= \frac{3m}{4} \frac{\Omega^3 f z}{v_{th}}$$

$$\Rightarrow \boxed{\frac{dm}{dt} = \frac{3}{4} \frac{\Omega^2 f}{v_{th}} Z m} \qquad (1)$$

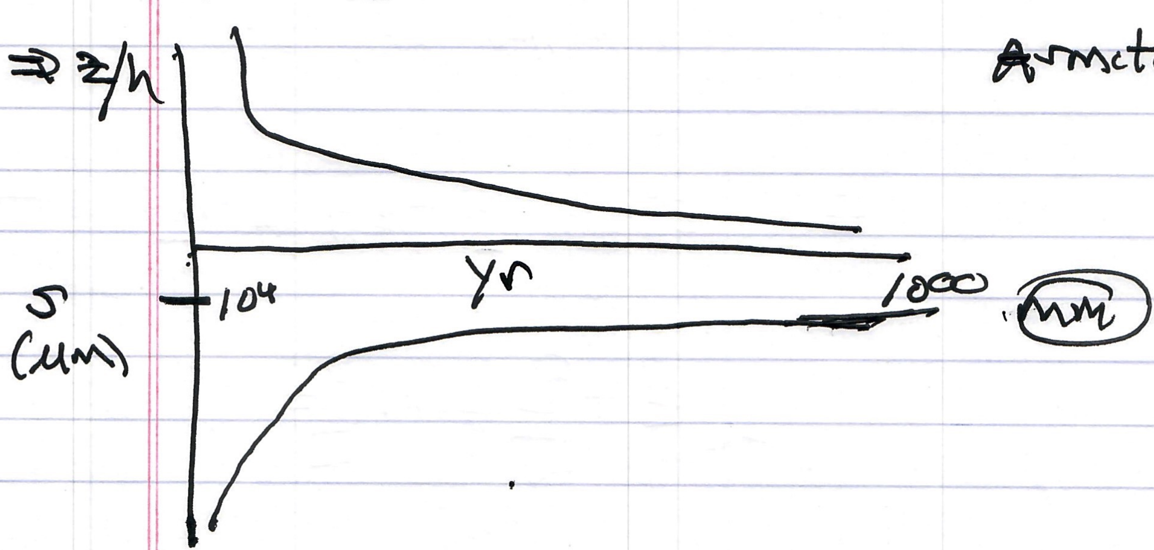
and

$$V_{settle} \sim \frac{\rho_m}{\rho} \frac{s}{v_{th}} \Omega^2 Z$$

$$\boxed{\frac{dZ}{dt} \sim - \frac{\rho_m}{\rho} \frac{s}{v_{th}} \Omega^2 Z} \qquad (2)$$

(1), (2) } basic model of particle growth and sedimentation in non-turbulent disk.

$\Rightarrow$



Armitage 4.1

Punch line: Grains grow to few mm → cm on midplane.

But → no turbulence!

→ Turbulence produces diffusion, as particle tightly coupled to gas, and gas should be turbulent (!?) for accretion.

N.B. Trade off!

Turbulent diffusion will inhibit particle settling to layer h if:

$$t_{diff} \sim h^2/D \quad < \quad t_{settle} \text{ for } z \sim h$$

$$t_{sett} \sim \frac{\Sigma}{\rho_m s \Omega}$$

$$\frac{h^2}{D} \leq \frac{\Sigma}{\rho_m s \Omega}$$

$\Rightarrow$

$$\boxed{D \geq \frac{\rho_m s h^2 \Omega}{\Sigma}}$$

If $D \sim \nu$ (but $\nu \to$ magnetic stress for MRI ! ?)

$$D \sim \nu \cong \alpha c_s^2/\Omega$$

$\Rightarrow$

$$\alpha > \frac{\rho_m s}{\Sigma}$$

if $\left.\begin{array}{l} \Sigma \sim 10^3\,gm/cm^2 \\ \rho_m \sim 3g\,cm^{-3} \\ s \sim 1\,\mu m. \end{array}\right\} \Rightarrow \underline{\alpha > 10^{-5}}$

→ small dust remains in suspension

$s \sim 1\,mm$

$\alpha \gtrsim 10^{-2}$ } larger particles will sediment.

Now, need more systemmatic treatment.

⇒

Fokker-Planck Theory for sedimentation in turbulence.

Sedimentation with Turbulence

Friction

Stokes drag to velocity of disk flow (turbulent)

$$\frac{dv_z}{dt} = -\gamma\left(v_z - \tilde{V}_z(x,t)\right) + \frac{\tilde{F}}{m_g}$$

buoyancy

Brownian force (Thermal noise

$$-g_z \frac{m_d - m_g}{m_d}$$

buoyancy - dust grains only

$$-g_z > 0$$

$$-g_z = \Omega^2 z$$

So

$$\frac{dv_z}{dt} = -\gamma\left(v_z - \tilde{V}_z(x,t)\right) - g_z \frac{\cancel{\rho_d}}{\cancel{\rho_d}}$$

Assume terminal velocity achieved, so $\frac{dv}{dt} = 0$

$$\frac{dz}{dt} = \tilde{V}_z(x,t) - \frac{g_z}{\gamma}\frac{\cancel{\rho_d}}{\cancel{\rho_d}}$$

$$= \tilde{V}_z(x,t) - \frac{\Omega^2 z}{\gamma}\frac{\cancel{\rho_d}}{\cancel{\rho_d}}$$

⇒ Langevin Equation:

$$\boxed{\frac{dz}{dt} = \tilde{V}_z(x,t) - \frac{\Omega^2 z}{\gamma}}$$

→ here turbulent velocity field sets random element.

dust density

Then seek $n(x, z, t)$. Proceed as in:

Schmoluchowski / Fokker-Planck equation:

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial z}\cdot\left\{ \underbrace{\frac{\langle \Delta z\rangle}{\Delta t}}_{\text{drift}}\, n - \frac{\partial}{\partial z}\underbrace{\frac{\langle \Delta z\Delta z\rangle}{2\Delta t}}_{\text{diffusion.}}\, n \right\}$$

see: { 210b Notes Fall '20
Lecture 6 a, b.

$$\left\langle \frac{\Delta z}{\Delta t}\right\rangle = -\frac{\Omega^2 z}{\gamma}$$

so

$$-\frac{\partial}{\partial z}\left[\left\langle \frac{\Delta z}{\Delta t}\right\rangle n\right] = +\frac{\partial}{\partial z}\left[+\frac{\Omega^2}{\gamma} z \; n\right]$$

vertical diffusion ↑

drift

$$\frac{\langle \Delta z \Delta z \rangle}{2\Delta t} = \int_0^t dt' \int_0^{t} dt'' \langle \tilde{V}_z(x, t') \tilde{V}_z(x, t'') \rangle$$

$$\cong \int_0^t dt_+ \int_{-\infty}^{\infty} dt_- \langle \tilde{V}_z^2(t_-) \rangle$$

$$= D_{zz} t$$

$$D_{zz} = \int_0^\infty \langle \tilde{V}_z(0) \tilde{V}_z(s) \rangle ds$$ → vertical diffusion coefficient

→ set by velocity correlation of underlying turbulence

In principle, $D_{zz} = D_{zz}(r, z)$

as velocity field is spatially dependent so,:

drag/drift — diffusion

$$\frac{\partial n}{\partial t} = +\frac{\partial}{\partial z}\left[\left\{\frac{\Omega^2 z}{\gamma}\, n\right\} + \frac{\partial}{\partial z} D_{zz}\, n\right]$$

advection – diffusion form.

Armitage, after Dubrulle, et. al '95 (posted), gives:

$$\frac{\partial \rho_d}{\partial t} = D\frac{\partial}{\partial z}\left[\rho\frac{\partial}{\partial z}\left(\frac{\rho_d}{\rho}\right)\right] + \frac{\partial}{\partial z}\left(\Omega^2 t_{visc}\, z\, \rho_d\right)$$

? ?

substantially equivalent, not quite identical..

This defines a dust layer scale:

$$\frac{\partial n}{\partial t} \sim 0 \Rightarrow$$

$$n \approx n(0)\exp\left[-z^2/2h_d^2\right]$$

$$1/h_d^2 = \frac{\Omega^2 \cancel{\rho}}{\gamma \cancel{\rho_g} D_{z,z}}$$

$$\boxed{h_d = \left[\left(D_{z,z}/\Omega^2 \right) \gamma \cancel{\frac{\rho}{\rho_d}} \right]^{1/2}}$$

- strange turbulence broadens the dust layer
- higher drag γ reduces drift
- if $\quad D_{z,z} \sim \alpha \, c_s H \qquad\qquad \gamma \sim D$

$$\sim \alpha H^2 \Omega \text{[redacted]}$$

$$\boxed{\frac{h_d}{H} \sim \left[\frac{\alpha \, \gamma \, \cancel{\rho/\rho_g}}{\Omega} \right]^{1/2}}$$

For concentration at disk midplane,

$$\boxed{\Omega T_\gamma \sim \Omega T_{drag} \gg \text{[redacted]} . \alpha .}$$

i.e for thin sediment layers

→ condition is that $\Omega t_{fric} \gg \alpha$

i.e. Dimensionless friction time $\gg \alpha$.

→ For realistic α, need substantial particle growth prior to settling.