Physics 239

Lecture 11 : Magnetic Buoyancy and Miscellaneous Magnetics

Comment - MRI Saturation - Mixing Length Model (δr cancelled).

→ Recall:

$\alpha_{SS} \cong 1/\beta^{\sigma}$; $\beta = C_s^2/V_{A0}^2 = \beta(r)$

$0 < \sigma < 1$

$\nu = \alpha_{SS} C_s h$

$\eta = ?$

[revisit stability]

- $\sigma \sim 1 \rightarrow \langle \tilde{B}_r^2 \rangle / 4\pi \rho_0 \sim V_{A0}^2$

  $\sigma \sim 0 \rightarrow \langle \tilde{B}_r^2 \rangle / 4\pi \rho_0 \sim C_s^2$

A useful way to formulate the MRI saturation problem:

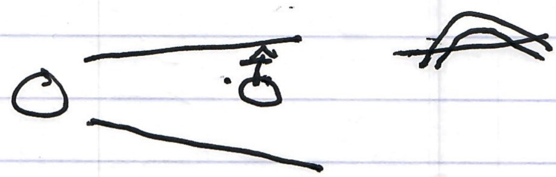
- What is σ? - explore range.
- Examining scaling exponent seems more relevant than $\alpha \sim .01, .001$ etc.

Analogue: MFE $D = D_B(\rho_0/L_\perp)^{\sigma}$

$0 < \sigma < 1 \rightarrow$ Gyro-Bohm breaking.

→ Fate of the Field?

→ Buoyancy ⇒ flux tube / loop rises to surface



→ $p + \frac{B^2}{8\pi} \sim \text{const}$ so

$\delta p + \frac{\delta B^2}{8\pi} \sim 0$ ⇒ high field slug naturally lower ρ

$p = p_0(\rho/\rho_0)^\gamma$ ⇒ rises like thermal plume heat

⇒ will decouple field from accretion disk (anchoring?)

Now:

i.) Theory of magnetic buoyancy $\left\{\begin{array}{l}\text{slab}\\ \text{disk}\end{array}\right.$

ii.) Comments on Parker Instability

iii.) Disk Coronae

iv.) Miscellaneous Magnetics ⇒ $\left\{\begin{array}{l}\text{Layered}\\ \text{Disk.}\end{array}\right.$

This brings us to $\begin{cases} \text{magnetic buoyancy} \\ \text{Parker instability} \end{cases}$

⇒ ① Vertical flux of magnetic field // intensity

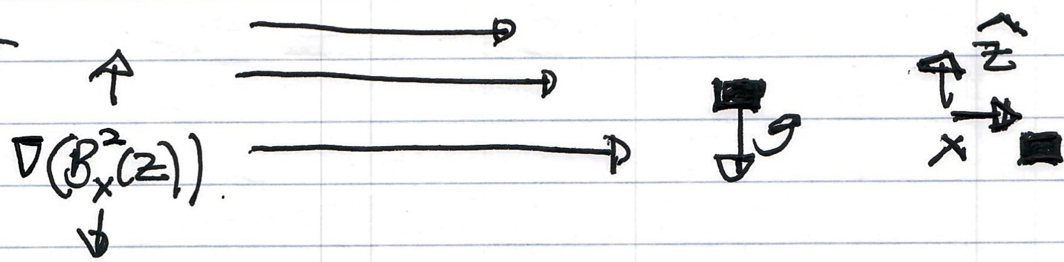
→ saturation mechanism for $\langle \tilde{B}^2 \rangle$

② Change of disk structure.

→ Basic Physics of Magnetic Buoyancy

→ Consider

slab



continuum.

$g > 0$

$$\rho_0 \left[\frac{\partial v}{\partial t} + \underline{v} \cdot \underline{\nabla}\, \underline{v} \right] = -g \tilde{\rho} \hat{z} - \underline{\nabla} p^* + \frac{\underline{B} \cdot \underline{\nabla}\, \underline{B}}{4\pi}$$

$$\frac{\partial \tilde{v}}{\partial t} = -g \frac{\tilde{\rho}}{\rho_0} \hat{z} - \frac{\nabla p^*}{\rho_0} + \cancel{\frac{B_{x0} \partial_x \tilde{B}}{4\pi}}$$

flute → $k_x = 0$.

no bending

Then, $\underline{\nabla}\cdot\tilde{\underline{V}} \cong 0$ $\Rightarrow$ buoyancy is slow, relative to magnetosonic wave

$\partial_z < k_\perp$

$$0 \cong -\frac{\underline{\nabla}\tilde{p}^*}{\rho_0} - g\,\cancel{\partial_z \frac{\tilde{\rho}}{\rho_0}}$$

$$\omega_{MS}^2 = c_s^2 + v_A^2$$

$\Rightarrow$ $\omega^2, N^2 \ll \omega_{MS}^2$

$\hookrightarrow$ magnetosonic wave

$\Rightarrow$ buoyancy slow, maintains perturbed pressure balance.

$$\tilde{p}^* \cong 0 \Rightarrow \boxed{\frac{\tilde{\rho}}{\rho_0} + \frac{\tilde{T}}{T_0} + \frac{\tilde{P}_M}{P_0}}$$

$$\boxed{\frac{\tilde{\rho}}{\rho_0} = -\frac{\tilde{T}}{T_0} - \frac{\tilde{P}_M}{P_0}}$$

$\hookrightarrow$ magnetic pressure perturbation driven cell

$\rightarrow$ B field as "stuff"

see: spiegel & weiss

$\Rightarrow$ Temperature perturbation, as usual.

How compute $\tilde{P}_M$? — Induction

$$\frac{\partial \underline{B}}{\partial t} + \underline{V}\cdot\underline{\nabla}\,\underline{B} = \underline{B}\cdot\underline{\nabla}\,\underline{V} - \underline{B}\,\underline{\nabla}\cdot\underline{V}$$ (ideal, simplicity).

$$\partial_t \tilde{\underline{B}} + \tilde{V}_z \partial_z B_x(z) = \cancel{B_x \partial_x \tilde{\underline{V}}} - B_x \underline{\nabla}\cdot\tilde{\underline{V}}$$ (flute)

Why the $\underline{\nabla}\cdot\underline{\tilde{V}}$?

$$\cancel{\frac{\partial \tilde{\rho}}{\partial t}} + \tilde{V}_z \frac{\partial \rho_0}{\partial z} = -\rho_0 \underline{\nabla}\cdot\underline{\tilde{V}}$$

(anelastic)

$\omega \ll \omega_s, \omega_{MS}$ → neglect acoustic coupling !

kc_s

but retain weak compression

$$\underline{\nabla}\cdot\underline{\tilde{V}} = -\frac{1}{\rho_0}\frac{d\rho_0}{\partial z}\tilde{V}_z$$ → anelastic approx

$\underline{B}\cdot$ ⇒

$$\partial_t \tilde{P}_M + \tilde{V}_z \partial_z \frac{B_x^2}{2} = +\frac{\partial \rho_0}{\partial z}\tilde{V}_z$$

∴

$$\boxed{\partial_t \tilde{P}_M = -B_0^2 \tilde{V}_z \frac{d}{dz} \ln \left| \langle B\rangle / \rho_0 \right|}$$

gradient drive → B field

comes from compression

and can retain resistivity, etc.

exercise – AD convection for magneto-convection

For temperature perturbation:

$$\frac{\partial \tilde{s}}{\partial t} + \tilde{V}_z \frac{\partial}{\partial z} s_0 = 0$$ isentropic

$$s = \ln\left[\frac{P}{P_0}(\rho/\rho_0)^{-\gamma}\right]$$

$$= \ln\left[T(\rho/\rho_0)^{-(\gamma-1)}\right]$$

usually
$\tilde{s} \to \frac{\partial s}{\partial \rho}\tilde{\rho} + \frac{\partial s}{\partial T}\tilde{T}$
and plug for $\tilde{\rho}$

but and recall:

$$\frac{\tilde{\rho}}{\rho_0} = -\frac{\tilde{T}}{T_0} - \frac{\tilde{P}_M}{P}$$

then as usual for convection:

$$\boxed{\partial_t\left[\tilde{T} + \frac{1}{c_p\rho_0}\tilde{P}_m\right] = -\frac{\tilde{V}_z T_0}{\gamma}\frac{d}{dz}\ln\left[P\rho^{-\gamma}\right]}$$

and, as usual, form equation for component vorticity along B_0. $\Rightarrow$ Overturning cell motion.

$$\underline{V} = \underline{\nabla}\phi \times \hat{x}$$

$$\Rightarrow \boxed{\partial_t \nabla^2 \tilde{\phi} = g\,\partial_y \left(\frac{\tilde{T}}{T} + \frac{\tilde{\rho}_m}{\rho_0} \right)}$$

and $\tilde{T}/T_0$, $\tilde{\rho}_m/\rho_0$ equations $\Rightarrow$

$$\underbrace{\gamma^2}_{\text{growth rate}} = \frac{-g k_y^2}{k_z^2 + k_y^2} \Bigg[\frac{\pm}{\underbrace{\gamma}_{\text{spec heat ratio}} P_0} B_0^2 \underbrace{\frac{d}{dz} \ln\left(\frac{\langle B\rangle}{\rho}\right)}_{\text{magnetic buoyancy}}$$

$$+ \frac{\cancel{\pm}}{T_0 \gamma} \frac{d}{dz} \underbrace{\left[\ln\left(P \rho^{-\gamma} \right) \right]}_{\text{thermal buoyancy}} \Bigg]$$

consider competition

$$= \underbrace{\gamma^2_{\text{conv.}}}_{\text{usual}} - \frac{g k_y^2}{k_z^2 + k_y^2} \left[\frac{\pm}{\gamma P_0} B_0^2 \frac{d}{dz} \ln\left(\frac{\langle B\rangle}{\rho}\right) \right]$$

$\to 1/B$

magnetic buoyancy drive

this all gives a generalized Schwarzschild criterion!

need $\frac{d}{dz} \ln \frac{\langle B\rangle}{\rho} < 0$

for contribution to instability

i.e. buoyancy instability criterion:
Counterpart of Schwarzchild

$$\boxed{\frac{d}{dz}\ln\left(P\rho^{-\gamma}\right) + \frac{B_0^2}{P_0}\frac{d\ln}{dz}\left(\langle B\rangle/\rho_0\right) < 0}$$

→ gives critical gradient of magnetic field relative to density gradient for instability (for adiabatic thermal)

$$\underline{\frac{1}{\langle B\rangle}\frac{dB}{dz} < \frac{1}{\rho_0}\frac{d\rho_0}{dz}}$$

and of course $\frac{dP}{dz} = -g\rho$ gives density profile. $(H = c_s/\Omega)$

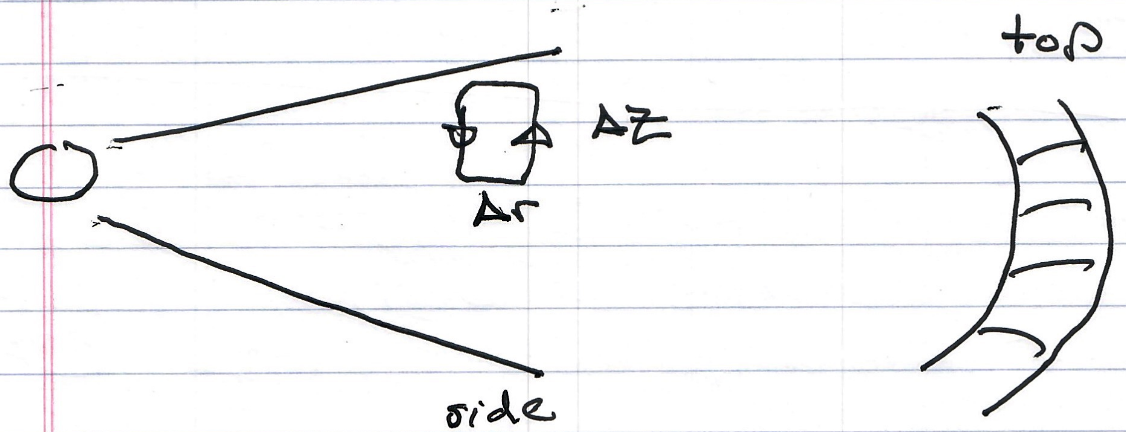
⇒ just as hot air, strong magnetic fields rise ..., and are lost from MRI process.

⇒ like thermal plumes, flux tubes/loops will rise to disk surface.

N.B.: – effects η, χ → β D
– staircase: Hughes + Brummell '21 Ap. J.

→ But this is a disk, not a slab...

As before, for buoyancy in disk:



→ consider axisymmetric rolls → else shearing limits buoyancy

→ vertical motion releases buoyancy potential energy

but radial motion costs energy, i.e.

$$\omega^2 = \frac{k_z^2}{k^2}\Phi \quad , \quad \text{as radial stratification}$$

stable.

∴ straightforward to show:

$$\omega^2 = \frac{k_r^2 N^2_{\substack{mag\\buoy}} + k_z^2 \Phi}{k_r^2 + k_z^2}$$

$$\left[\begin{array}{l} N^2_{\substack{mag\\buoy}} = +g\left[\dfrac{B_0^2}{P_0}\dfrac{d}{dz}\ln\left(\dfrac{\langle B\rangle}{\rho}\right) + \dfrac{1}{\gamma}\dfrac{d}{dz}\ln\left(P\rho^{-\gamma}\right)\right] \\ \Phi = \dfrac{2\Omega}{r}\dfrac{d}{dr}\left(r^2\Omega\right) \end{array}\right.$$

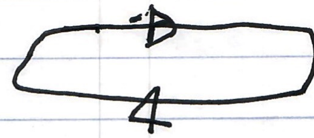
$$g \sim \Omega^2 h$$

or

$$\gamma^2 = \frac{\underbrace{(\Delta z^2)\left(-N^2_{\substack{mag\\buoyancy}}\right)}_{\text{gain}} - \underbrace{(\Delta r)^2 \Phi}_{\text{energy penalty}}}{(\Delta r)^2 + (\Delta z)^2}$$

analogue.
to
(elevators;
cf. Taylor-
Proudman)

⇝ favors thin, tall cells

i.e.  **not** 

⇒ thin loops at surface → limited coupling.

→ angular momentum transport inward

i.e. ~~[scribble]~~ $\Gamma_{L_\theta} \sim -D\,\partial_r(r^2\Omega)$

(show) $D \sim \langle \tilde{v}^2 \rangle / \gamma$

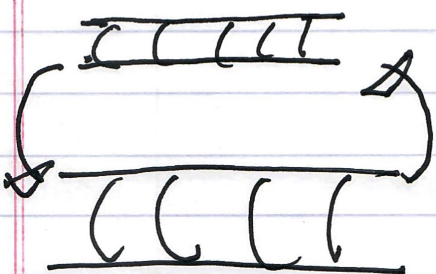
→ (magneto) convection not effective for viscosity

⇒ Magnetic buoyancy results in rise of magnetic loops to top (bottom) of disk. → magnetized corona

→ <u>Parker Instability</u> → variant

– Buoyancy, as described is an <u>interchange</u>

i.e.



i.e. conserve flux

$\underline{k}\cdot\underline{B} = 0$ → no bending, $\delta W_{\varphi^2} = 0$.

$$\delta W_{\varphi} = \int d^3x \left[\left(\cancel{\mathbf{B}_0\cdot\nabla\xi} - \underline{\xi}\cdot\nabla\mathbf{B} - \mathbf{B}\,\underline{\nabla}\cdot\underline{\xi} \right)^2 \right]$$

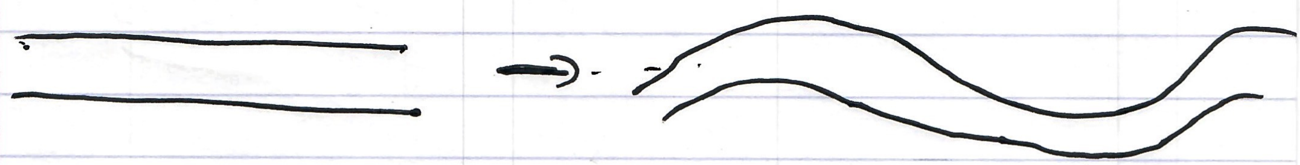
flute

$$\underline{\nabla}\cdot\underline{V} = -\frac{1}{\rho}\frac{d\rho}{dt}$$

– Parker Instability:

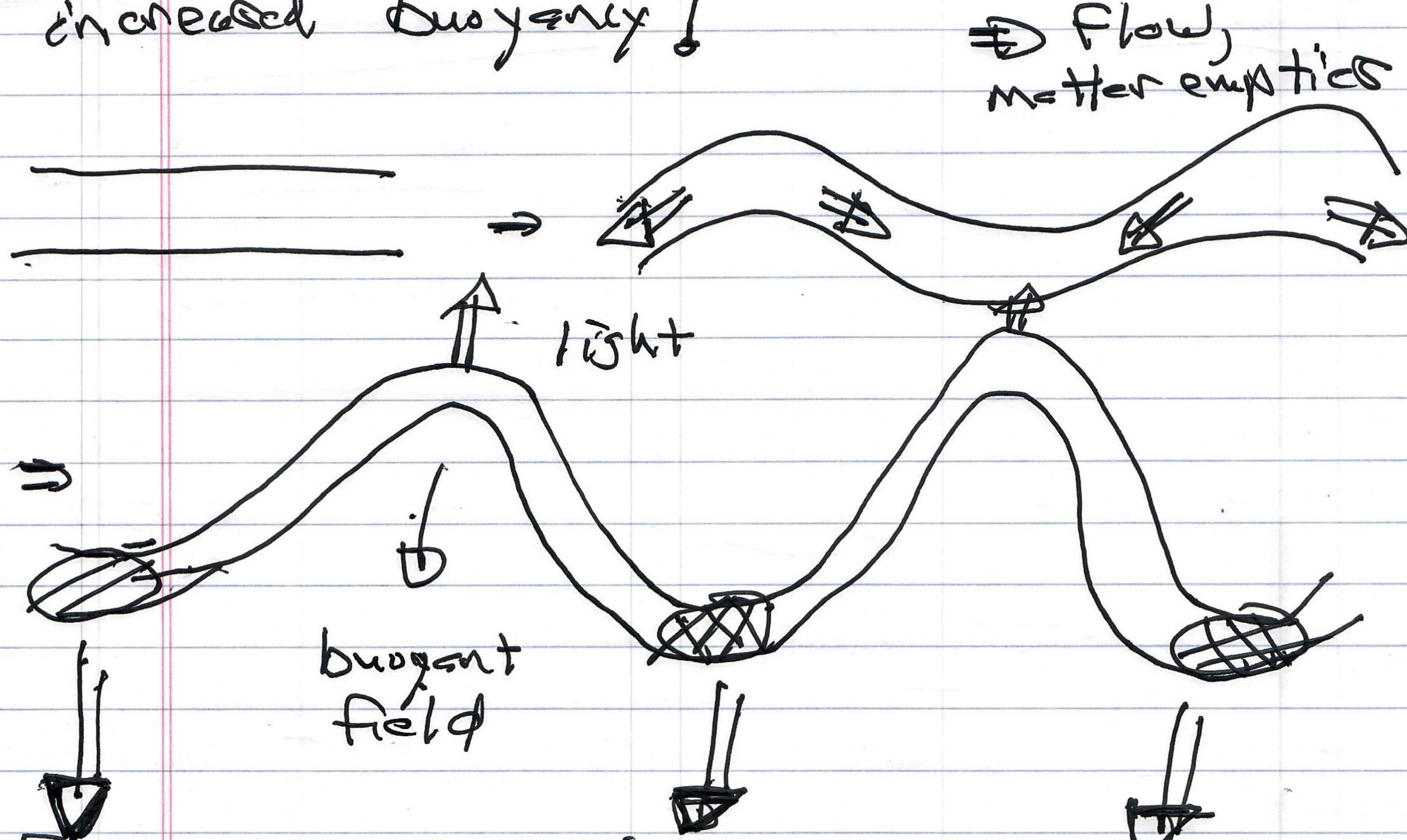
→ <u>mild bending</u>

i.e. How, why? → seems energetically unfavorable.



but:

having bent flux tube, matter can slide/flow along B,
⇒
increased buoyancy!



mass concentrations, produced by flow and compression

→ key point: Energy gain by sliding/flow producing mass concentrations outweighs penalty due increased bending.

→ Can explain emergence of bent or "buckled" field (Shu).

→ Process is <u>nonlinear</u>, fundamentally!

<u>i.e.</u> Finite amplitude perturbation produces flow via gravity → sliding

⇒ reinforces perturbation, by draining tube.

→ Linear instability exists but connection to physical picture above is <u>obscure</u>. Involves different competition of terms in δW..

→ Parker instability in disk will tend to low k_r, low k_ϕ

i.e. will bend azimuthal field. Latter inevitable, due $\tilde{B}_r$ + V_ϕ'.

→ Disk Coronae – why?