→ Observe also:

- Dissipated power/area $= D(r)$

$$= \nu \Sigma (r\Omega')^2$$

so, in annulus $2\pi r \Delta r$ ⇒

→ Dissipated Energy $= 2\pi r D(r) \Delta r$

$$= \frac{3}{2} \frac{G M \dot{M}}{r^2} \Delta r$$ as above

[recall $3\pi \nu \Sigma = \dot{M}\left(1 - (r_{in}/r)^{1/2}\right)$]

and

L_{disk} ⇒ Grav Energy Released $= \frac{G M \dot{M}}{2 r^2} \Delta r$

So: - 1/2 ⇒ Grav energy released

- 3/2 ⇒ dissipated energy

Difference? $\Rightarrow$ energy transport

recall $P = -\left(\frac{\partial}{\partial r}(G\Omega) - G\Omega'\right)\Delta r$

$\downarrow$

accounts for rest.

$\rightsquigarrow$ Temperature?

$D(r)$ = power / area $= \nu\Sigma(r\Omega')^2$

related $\dot{M}$

so for black body:

$D(r) = 2\sigma_{SB} T_s^4$

$\downarrow$ 2 sides

$\hookrightarrow$ surface temperature

$\boxed{\text{equate dissipated power with Stefan-Boltzmann}}$

so

$$T_s^4 = \frac{3}{8\pi}\frac{GM\dot{M}}{\sigma_{SB}\, r^3}\left(1 - \left(\frac{r_{in}}{r}\right)^{1/2}\right)$$

$$\boxed{T_{disk} = \left(\frac{3}{8\pi} \frac{G M \dot{M}}{\sigma_{SB}\, r_{un}^3} \right)^{1/4}}$$

- no explicit r dependence!
- implicit via $\dot{M}$

See Pringle, Lodato for Spectral Energy Distributions

→ Time Scales – Aposteriori

Its convenient now to review time scales and their ordering.

Recall – used velocity scales to construct time scale ordering

– simplified transport equations to obtain V_r, mass diffn.

Time Scales :

① - dynamical - rotation

② - vertical

③ - thermal - cooling

④ - viscous - Functions of radius

so

$$① \sim \Omega^{-1} \sim \left(r^3/GM\right)^{1/2}$$

$$② \sim H/c_s \sim \Omega^{-1} \sim \left(r^3/GM\right)^{1/2}$$

note $\tau_{rot} \sim \tau_{vert}$.

③ Cooling.

$$\tau_{cooling} \sim \frac{\text{Thermal Energy Density}}{\text{Viscous Heating rate}}$$

→ necessarily, at equilibrium.

Thermal Energy Density $\sim P/(\gamma-1)$

$$P/P_0 \sim (\rho/\rho_0)^\gamma$$

then: $\begin{cases} TED \sim \Sigma\, c_s^2/\gamma(\gamma-1) \\ c_s^2 = dP/d\rho \end{cases}$

Heating Rate : $\nu \Sigma (r\Omega')^2$

so

$$T_{cooling} \sim \frac{\Sigma c_s^2/\gamma(\gamma-1)}{\nu \Sigma (r\Omega')^2}$$

if $\nu = \alpha \Omega H^2$

$= \alpha c_s H$

α 'captures' microphysics

$\downarrow$ extensive discussion coming

$$T_{cooling} \sim 4/9\gamma(\gamma-1)\,\alpha\,\Omega$$

(4) viscous

$$\tau_{visc} \cong r^2/\nu .$$

and have ordering:

$$\boxed{\tau_{dyn} \sim \tau_{vert} < \tau_{th} < \tau_{visc}}$$

N.B.: Disk accretes slowly, maintains thermal equilibrium, with quasi-static structure.

NB: - All time scales r dependent

- Outer disk evolves more slowly.

→ Disk Outbursts and (Transport) Instabilities.

→ Have extensively discussed disk structure on foundation:

$$T_{r,\phi} = \Sigma r \nu \frac{\partial \Omega}{\partial r}$$

ν → viscosity

→ Have said zip about (structure of dependencies) ν,

→ Is this house-of-cards internally consistent

→ Σ dependence.

Now, recall:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r}\left[r^{1/2} \frac{\partial}{\partial r} \left(r^{1/2} \nu \Sigma \right) \right]$$

$$\left(\dot{M} = -2\pi r \overbrace{v_r}^{\sim \nu} \Sigma \right)$$

ν^* $\Rightarrow$ $\mu = \nu \Sigma$

$$\boxed{\frac{\partial \mu}{\partial t} = \frac{\partial \mu}{\partial \Sigma} \frac{3}{r} \frac{\partial}{\partial r}\left[r^{1/2} \frac{\partial}{\partial r}\left(\mu r^{1/2}\right)\right]}$$

- diffusion, again

$\frac{\partial \mu}{\partial t} \approx$ [redacted] $\nabla \underbrace{\left(\frac{\partial \mu}{\partial \Sigma}\right)}_{\text{schematic}} \nabla \mu$

"D" $\sim \frac{\partial \mu}{\partial \Sigma}$ sign ?

$\Rightarrow$ negative diffusion ?

$\Rightarrow$ instability

$\partial \nu / \partial \Sigma$ is critical!

⇒ Modelling (c.f. Bell, Lin '94) suggests →

→ S-curve!

→ bifurcation and back-transition

⇒ L.C.O ⇔ Disk outbursts / cycles

TBC.