Physics 239 Disks + Dynamics W'22

Lecture 1

- Handouts, Problem Set 0
- Gaseous Disks - mostly {proto-stellar, proto-planetary}
- Some basic ideas, scales, ...
- Basics of viscous flow in cylindrical geometry..

Physics 239 **Disks and Dynamics** **Winter 2022**

Course Topics

1) **Overview of Disks; Accretion, Galactic Structure**

2) **Keplerian Disks and Accretion: Basics**

Part I: Accretion

i) Structure, equilibrium profiles
ii) Angular momentum transport, alpha viscosity concept and prescription
iii) Mechanism: Axisymmetric Interchange (Rayleigh), Two Particles (Lynden–Bell and Pringle) and the end state
iv) Convection: axisymmetric, non-axisymmetric

3) **Magnetic Fields and Accretion**

– mostly gaseous, fluid
– B-field essential

i) Essentials of MHD
ii) MRI (magneto-rotational instability), MRI—Lynden–Bell connection, mixing length estimates of alpha
iii) Disk Dynamos — An Introduction
iv) Fate of the field? — Parker Instability and Disk Coronae
v) Accretion in collisionless disks (AGNs)

4) **Protoplanetary Disks**

– Cooler, weak ionization

i) MRI for cooler, weakly ionized disks (effects: resistivity, ambipolar diffusion …)
ii) Convection revisited
iii) Introduction to planet formation in Disks
iv) Disk–Planet Interaction

– Particulate matter disks

5) **Self-Gravitating Disks and Galactic Dynamics I**

Part II: Galactic Dynamics

i) OV, stellar dynamics, Vlasov–Poisson System, Jeans Equations
ii) Stellar Orbits
iii) Basic ideas, Jeans and Toomre criteria
iv) BGK Solutions — stationary states
v) Violent Relaxation (Lyndell–Bell)
vi) Collisionless Jeans instability, ~~Landell~~ Landau Damping

– mostly collisionless

– stellar dynamics (ensemble of stars-as-particles)

– uses B+T

6) Self-Gravitating Disks and Galactic Dynamics II

i) Energy Principle for self-gravitating matter
ii) Spiral Waves
iii) Spiral Wave Amplification: Wave Kinetics for Spirals
iv) Galactic Magnetic Fields
~~v) Resistivity Angular Momentum Transport~~

7) Revisiting Angular Momentum Transport

TBD

3.

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Texts and References

Recommended Texts

i) P. Armitage, "Astrophysics of Planet Formation" — see ref. material

Good general text on Keplerian accretion disks, with an emphasis on planet formation.

* ii) J. Binney and S.D. Tremaine, "Galactic Dynamics"

Rather encyclopedic text on galactic dynamics. Widely known and used.

* iii) F. Shu, "The Physics of Astrophysics, Vol. 2: Gas Dynamics"

Good basic text on Astrophysical Fluids. Good treatment of accretion and spiral waves.

iv) S. Galtier, "Modern Magnetohydrodynamics"

Basic text on MHD. Good discussion of magneto-rotational instability.

+ copious reference material, posted.

Books referred in lecture notes.

4.

Physics 239 | **Disks and Dynamics** | **Winter 2022**

Course Syllabus

Fun Part[illegible] ?

Instructor: Patrick H. Diamond
Office location: SERF 436
Phone: (858) 534-4025
Email: diamondph@gmail.com (best to use this!); pdiamond@ucsd.edu
Office Hours: Open, *but* best to call or email first. Office hours will be by Zoom.

Lectures: Tuesday, Thursday 11:00 a.m. – 12:20 p.m.
SERF 329

Lecture notes and supplementary materials available online at:
https://canvas.ucsd.edu/courses/33821/

Discussions: TBD — on Zoom
Scheduled as needed.

Grades	Letter Grade	S/U
	40% Project 40% Notes 20% Participation	50% Project 50% Participation

Content: This course focuses on disks and their dynamics. We will discuss the structure and dynamics of both Keplerian and self-gravitating disks. We will emphasize accretion, implications for planet formation along with galactic structure, especially spirals.

This course will evolve into a new Astrophysics course on dynamics complementary to existing courses on Galaxy Formation and Astrophysical Fluids.

Background: A strong background in basic classical physics is required. Some familiarity with fluid dynamics is helpful.

See Problem Set Zero.

- Project: Paper on topic loosely related to course
- Notes: Value-added write-up of lectures

Part 1 : Gaseous Disks and Accretion

9a

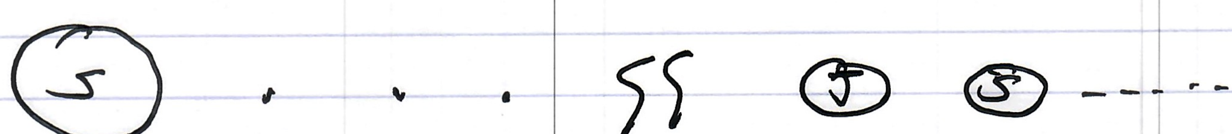
→ In a word, the focus of Part 1 is

Accretion.

→ The problem of accretion: disks → ubiquitous

- consider solar system
- planarity of solar system ⟺ evolved from star forming disk (→ Sun).

i.e. (Kant, Laplace → present)



form stars, planets etc

① bservations:

①— where did the mass end up?

→ $\boxed{\text{Sun}}$

$M_\odot \gg$ everything else

$M_\odot \sim 2 \times 10^{33}$ grams

②— where did angular momentum end up?

$L_\odot \cong M_\odot R_\odot^2 \Omega \sim 10^{49}\ g\ cm^2/sec$

$L_J \cong 2 \times 10^{50}\ g\ cm^2/sec$

+ Saturn, Uranus, Neptune ...

→ $\boxed{\text{Giant Planets}}$

∴ essence of accretion:

(i) – segregation of mass and angular momentum

mass – infall

angular momentum – outflow

(ii) – process of angular momentum transport to periphery, enabling mass inflow.

$$\boxed{\text{Central question in accretion is transport of angular momentum}}$$

and:

– natural way to "think" of angular momentum transport is viscous stress

– but 'collisional viscosity' here is feeble

$\nu \sim v_{th} \, l_{mfp} \sim v_{th} / n\sigma$?

- 'viscosity' here is due turbulent mixing

$$\nu = \underbrace{\alpha}_{\text{Shakura-Sunyaev parameter}} \underbrace{c_s H}_{\text{normalization}}$$

$v_{th} \to \tilde{v}$

$l_{mfp} \to l_{mix}$

- but Keplerian disks are remarkably stable, in gross hydrodynamic sense.

- { - turbulent viscous accretion on macro-stable disks ?!
  - origin of viscosity and underlying turbulence ?

→ { magnetic fields .....
  MHD physics ⇒ MRI

→ from disks to planets ....

(b.) Overview of Disks

- Disks ubiquitous {
  - Galaxies – collisionless
  - stars – gaseous
  - Planets & Rings – particles, granular flow

focus:

- Mostly stellar accretion disks

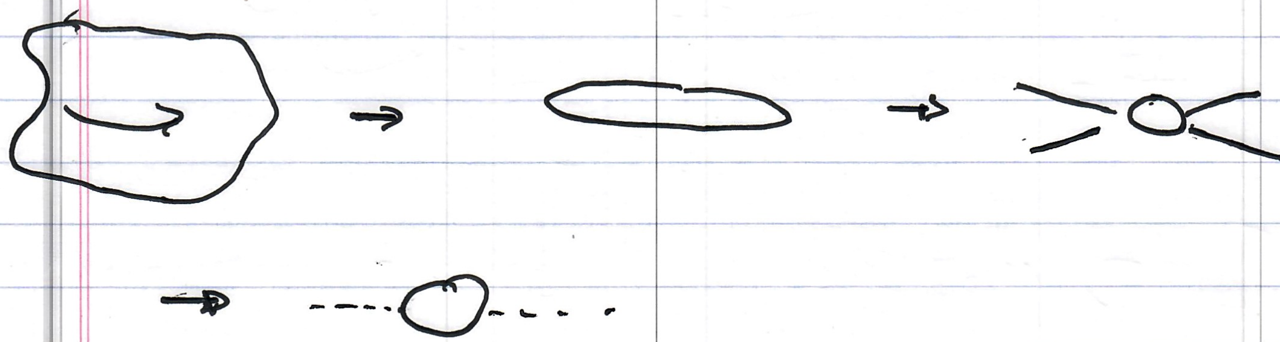
  i.e. disk feeds central object thru (viscous) angular momentum transport

- Why disks? – Essential part of star formation process.

  i.e. GMC → Disks → Star (+ planets)

  GMC = Giant molecular cloud.

Gas cloud on scale kpc, condenses under gravity ⇒ Molecular cloud core (→ disk + star).



Core ?

- scale : Jeans length!

i.e. $\omega^2 = k^2 c_s^2 - 4\pi G\rho$

$\omega^2 = 0 \rightarrow l_J \sim c_s / \sqrt{G\rho}$

mean molec. weight

$c_s^2 = k_b T / \mu m_p$

proton mass

$\therefore\ l_J \sim .1$ pc for $M \sim M_\odot$

- Mass

$M_J \sim l_J^3 \rho \sim c_s^3 / G^{3/2} \rho^{1/2}$

→ Jumping the gun .....

Compression → Spin-up

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\underline{\nabla}\,\underline{v}\right) = -\underline{\nabla}P$$

$$\frac{\partial \underline{v}}{\partial t} = \underline{v}\times\underline{\omega} - \frac{\underline{\nabla}P}{\rho}$$

if $P = P(\rho)$

$$\nabla\times\left(\frac{\underline{\nabla}P}{\rho}\right) = 0$$

$$\partial_t\,\underline{\omega} = \underline{\nabla}\times\underline{v}\times\underline{\omega}$$

↓

vorticity

$$\partial_t\,\underline{\omega} = -\underline{v}\cdot\underline{\nabla}\,\underline{\omega} + \underline{\omega}\cdot\underline{\nabla}\,\underline{v} - \underline{\omega}\,\underline{\nabla}\cdot\underline{v}$$

$$\underline{\nabla}\cdot\underline{v} = -d\rho/dt$$

$$\frac{d}{dt}\frac{\underline{\omega}}{\rho} = \frac{\underline{\omega}}{\rho}\cdot\underline{\nabla}\,\underline{v}$$

$\underline{\omega}/\rho$ frozen onto flow.

Collapse $\Rightarrow \rho \uparrow \Rightarrow \underline{\omega} \uparrow$

angular momentum disposal ?

— Angular momentum

$L_{core} \sim \Omega \, l_J^2 \, M_{core}$

$\Omega_{core} \sim 10^{-13} \rightarrow 10^{-14} \text{ sec}^{-1}$

(observation) $\boxed{\text{Physics ?}}$

$L_{core} \sim \Omega_c \left(C_s^3 / G\rho \right) \rho \, l_J^3$

so

$j \sim \Omega_{core} \, l_J^2 \sim 10^{21} - 10^{22} \text{ cm}^2/\text{sec.}$

$\downarrow$

<u>specific</u>
angular momentum
(per particle)

$L_{core} \sim M_{core} \, j$, for $M_\odot \sim 10^{33} \text{ g.}$

<u>far</u> exceeds angular momentum of sun

So → where does the angular momentum go?

Hint: Perfect accretion limit:
$M \to M_{star}$
+
1 particle at <u>large</u> radius.

⇒ Transport (viscous) outward, on disk.

– Profile:

Keplerian: $\frac{GM}{r^2} m = m \frac{v_\theta^2}{r}$

$$\Omega^2 \sim GM/r^3$$

$$v_\theta \sim (GM/r)^{1/2}$$

Disk size:

$$j \sim (GMr)^{1/2}$$

$\therefore \quad R_{disk} \sim j^2/GM$

$\sim 10^2 - 10^4$ AU.

but $R_{disk} \gg R_{star}$

so need accrete (lots) more

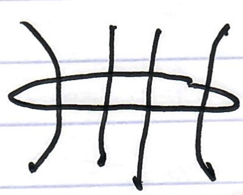
$\Rightarrow$ angular momentum transport needed!

Also: - Binaries

- B-fields $\Rightarrow$ magnetic braking.

$\vdots$ $\Rightarrow$ disk field configuration.

i.e. vertical thread azimuthal $B_\theta(r)$



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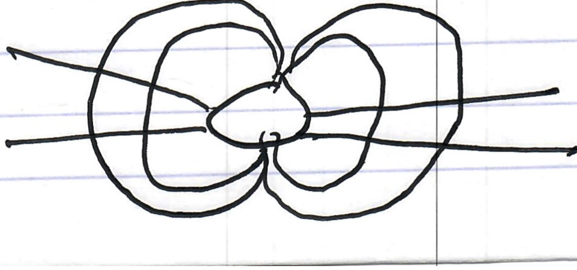


$B_z(r)$

→ Some Disk Properties:

- form around wide variety of stars

  $.1\, M_\odot$ to $10\, M_\odot$

- disk masses — fraction $M_\odot$
  ↓
  wide variety.

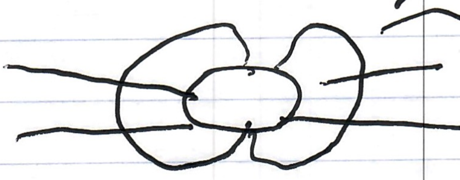
- $R \sim 10^4 - 10^5$ AU

- $\underbrace{H}_{\text{thickness}}/R \sim .1$

- Accretion FOM

  $\dot{M}$ → accretion rate (to central object)

  → measured by emission lines → stellar magnetosphere.



typical: $\dot{M} \sim 10^{-5} - 10^{-9}\, M_\odot / yr$

$\dot{M}/M \rightarrow$ rate

– Lifetimes $\sim$ 1-10 million years.

Next: Basic Disk Models, Hydrodynamics

Background

→ Viscous Hydro – Cylindrical Geometry
(see Landau Lifshitz "Fluids")

$$\partial_t (\rho \underline{v}) = - \underline{\partial_x} \cdot \underline{\underline{\Pi}}$$

$$\partial_t (\rho v_i) = - \frac{\partial}{\partial x_k} \Pi_{ik}$$

momentum balance

$$\Pi_{ik} = - \sigma_{ik} + \rho v_i v_k$$

σ_{ik} → stress tensor (thermal motion)

$\rho v_i v_k$ → Reynolds stress tensor (bulk motion)

$$\sigma_{i,k} = - p \, \delta_{i,k} + \sigma'_{i,k}$$

p → pressure

$\sigma'_{i,k}$ → viscous stress tensor → momentum flux not due direct fluid/mass motion.

⇒

$$\partial_t (\rho v_i) = - \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_k} (\rho v_i v_k) + \frac{\partial}{\partial x_k} \sigma'_{i,k}$$

For viscous stress:

→ expect $\sigma' \sim \frac{\partial v}{\partial x}$ (first deriv. only) (sliding T↑)

⇒ expansion in: $\underbrace{l_{mfp}/L_v}$ (lowest order)

→ no viscous stress for solid body rotation

hence $\sigma'_{ik} \sim \frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}$

vanish for $\underline{v} = \Omega \times r$

so most generally:

viscosity → shear

$$\sigma'_{ik} = \eta\left[\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right]$$

$$+ \zeta\delta_{ik}\frac{\partial v_l}{\partial x_l}$$

second (compressive)

so → Navier-Stokes Eqn. (after continuity):

$$\rho\left[\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\underline{\nabla}\,\underline{v}\right] = -\underline{\nabla} p + \eta \nabla^2 \underline{V}$$

$$+ \left(\zeta + \frac{\eta}{3}\right)\underline{\nabla}\,\underline{\nabla}\cdot\underline{v}$$

$$\underset{\substack{\downarrow \\ \text{kinematic viscosity (momentum diffusivity)}}}{\eta = \rho\,\nu}$$

$$\underline{\nabla}\cdot\underline{v} = 0$$

$$\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\underline{\nabla}\,\underline{v} = -\frac{1}{\rho}\underline{\nabla} p + \nu \nabla^2 \underline{V}$$

$$\underline{N\text{-}S}$$

Now, in cylindrical geometry:
Continuity:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}}_{=0} + \frac{1}{\rho}\left[\partial_t \rho + \underline{v}\cdot\nabla\rho\right]$$

$$\nabla\cdot\underline{V} = 0$$

N-S

cent

$$\frac{\partial v_r}{\partial t} + \underline{v} \cdot \underline{\nabla}\, v_r - \frac{v_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu \left[\nabla^2 v_r - \frac{2}{r^2}\frac{\partial v_\phi}{\partial \phi} - \frac{v_r}{r^2} \right]$$

$$\frac{\partial v_\phi}{\partial t} + \underline{v} \cdot \underline{\nabla}\, v_\phi + \frac{v_r v_\phi}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \phi} + \nu \left[\nabla^2 v_\phi + \frac{2}{r^2}\frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r^2} \right]$$

$$\frac{\partial v_z}{\partial t} + \underline{v} \cdot \underline{\nabla}\, v_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \nabla^2 v_z \quad \checkmark$$

where:

$$\underline{v} \cdot \underline{\nabla} = v_r \frac{\partial}{\partial r} + \frac{v_\phi}{r}\frac{\partial}{\partial \phi} + v_z \frac{\partial}{\partial z}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

and viscous stresses:

(other components straightforward).

$$\sigma_{r,\phi} = \eta\left(\frac{1}{r}\frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right)$$

radial transp ϕ mom.

for $v = v_\phi(r)$ disk

$$= \eta\left(\frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right)$$

vanishes for solid body!

$$v_\phi = \Omega r, \qquad \Omega = \Omega(r)$$

$$\boxed{\sigma_{r,\phi} = \eta\, r\, \frac{\partial \Omega}{\partial r}} \longleftrightarrow$$

Ω' drives viscous stress

and

$$\frac{\partial}{\partial t} v_\phi + \dots = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\phi}{\partial r}\right) - \frac{v_\phi}{r^2}\right]$$

$$= \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(r\Omega)\right) - \frac{\Omega}{r}\right]$$

$$= \nu\left[3\Omega' + r\Omega''\right]$$