Relativistic magnetic reconnection in laser-driven plasma

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Abstract

Magnetic reconnection is an ubiquitous phenomenon in many plasma system such as gamma-ray bursts and active galactic nuclei. In the magnetic reconnection, the magnetic filed lines will reconfigure and the magnetic energy will be converted to the kinetic energy. We reviewed the reconnection model in non-relativistic and the relativistic case and discussed some reconnection experiments using lasers. In the non-relativistic case, the Sweet-Parker model and the non-MHD model are investigated. We also studied the relativistic Sweet-Parker model. The relativistic model shows that the inflow velocity is proportional to the magnetic Reynolds number as $R_M^{-1/2}$, which is the same as the non-relativistic case. The reconnection can be produced in laboratory with two laser focusing on adjacent spots, giving an opposite magnetic field in the middle of the two spots. Some examples of the non-relativistic and relativistic reconnection using lasers are reviewed.

1 Theory and background

Magnetic reconnection is a common phenomenon in space where the magnetic field lines are topologically changed and the magnetic energy is converted to the kinetic energy. It is observed in solar wind interacting with the magnetosphere, solar flares, etc. When the two opposite magnetic field lines are close to each other, the topology of the field lines will change due to the finite resistivity. In this section, we will first review the Sweet-Parker model, which is based on MHD formulation. And then we will talk about the non-mhd model. Last bust not least the relativistic reconnection model will be introduced.

1.1 Sweet-Parker model

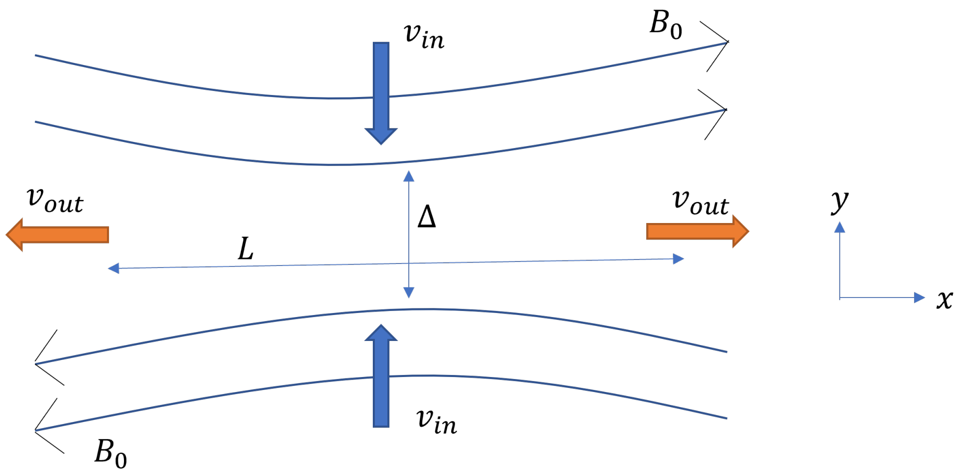

Figure 1: Configuration of Sweet-Parker reconnection.

In the Sweet-Parker model, we use the mass, momentum and energy conservation. First we look at the mass conservation. Assuming the incompressible fluid, we have $\nabla \cdot v = 0$. So the inflow mass is equal to the outflow mass, we get

$$\rho_0 v_{in} L = \rho_0 v_{out} \Delta. \tag{1}$$

This gives the relation between v_{in} and v_{out} as $v_{in} = v_{out}\Delta/L$.

We then use the conservation of momentum in the x direction along the current sheet. From the momentum equation, we can rearranege it and will get $p + B_0^2/8\pi + \rho_0^2 v/2 =$ constant. In the middle of the current sheet, the velocity is zero and the magnetic field is finite. On the other hand, at the end of the current the velocity is v_{out} and the magnetic field goes to zero. We have

$$p + \frac{B_0^2}{8\pi} = p + \frac{\rho_0 v_{out}^2}{2}. \tag{2}$$

This gives that $v_{out} = v_A$,where $v_A = \sqrt{B_0^2/(4\pi\rho_0)}$ is the Alfven speed. So we get

$$v_{in} = v_A \frac{\Delta}{L}. \tag{3}$$

Next we use the energy balance: the rate of inflow magnetic energy equal to the rate of Ohmic dissipation. The Ohmic dissipation power is

$$P_{OH} = \frac{J^2}{\sigma}\Delta L = (\frac{c}{2\pi})^2 \frac{B_0^2}{\Delta^2}\frac{L\Delta}{\sigma}, \tag{4}$$

where Ampere's law is used. The magnetic energy flow power is

$$P_{in} = 2\left(\frac{B_0^2}{8\pi}\right) vL. \tag{5}$$

By equating the above two, we can get $v_{in} = \eta/\Delta$, where $\eta = c^2/(\pi\sigma)$. Finally with $v_{in} = v_A \Delta/L$, we will get

$$\frac{\Delta}{L} = R_M^{-1/2}, \quad \text{and} \quad v_{in} = v_{out} R_M^{-1/2}, \tag{6}$$

where $R_M = v_A L/\eta$ is the magnetic Reynolds number.

We arrived that the reconnection time from Sweet-Parker model is

$$\tau_{SP} = L/v_{in} = (L/v_A) R_M^{1/2}. \tag{7}$$

The reconnection time is much longer than the Alfven time $\tau_A = L/v_A$, but much shorter than the resistive decay time in macroscopic scale L, $\tau_{decay} = L\,S/v_A$. According to the observation data in solar flare [1], the reconnection time of Sweet-Parker model is $\tau_{SP} \sim 10^8$ seconds. However the observed energy release time is about 10^3 to 10^4 seconds. There is a huge discrepancy. There are also other models trying to explain this, such as Petschek's Model using shock explanation. We will discuss the non-MHD model in the next subsection.

1.2 Non-MHD model

We so far consider the reconnection in resistive MHD formulation. It is necessary to check the validity of the MHD. Take the example of the solar flare. The relative drift velocity of electron and ion is

$$\Delta v = v_i - v_e = \frac{c\,j}{n\,e} = \frac{c}{n\,e}\frac{B_0}{4\pi\Delta}. \tag{8}$$

Using the layer width Δ from Sweet-Parker model, we have $\Delta = L/\sqrt{R_M}$. Given the parameters of solar flare that $L = 10^9$ cm, $R_M = 2.7 \times 10^{12}$, $B_0 = 300$ G, $\rho = 10^{-12}$ g/cm^3, we can get the relative drift velocity of ion and electron is $\Delta v = 2.5 \times 10^9$ cm/s [1]. Also the ion sound speed is about 10^7 cm/s for $T = 10^2$ eV. We will expect an ion acoustic instability to happen for such large relative drift velocity. Thus, the Sweet-Parker model fails since the relative drift is large enough to drive the ion acoustic instability, which will make resistivity larger.

The validity of the resistive MHD depends on the comparison between the ion inertial length c/ω_{pi} and the thickness the the reconnection layer Δ. The resistive MHD is valid when

$$\Delta > \frac{c}{\omega_{pi}\sqrt{\beta}}, \tag{9}$$

where $\beta = 8\pi p/B^2$. Here p is the plasma pressure in the current sheet and the B is the magnetic field outside the current sheet. And we also have

$$\frac{\Delta v}{v_i} = \frac{c}{\omega_{pi}\Delta\sqrt{\beta}}. \tag{10}$$

So if we need resistive MHD and Sweet-Parker model to be valid, we will have

$$\Delta v \ll v_i. \tag{11}$$

This is also the situation when ion-acoustic instability cannot occur.

The large relative drift velocity, $\Delta v \gg v_i$ will lead to the ion acoustic instabilities. Then the ion acoustic waves will drag ion and electron toward the wave phase velocity. This force from wave-particle interaction will cause larger effective resistivity. Larger resistivity will result in larger layer width Δ and larger reconnection speed v_{in}. From Eq. 10, we can see that the larger Δ will give smaller Δv. As the Δv gets smaller than the critical relative drift velocity, there will be no instablity. We can get the critical layer width Δ_c by equating Eq. 10 to 1, giving that

$$\Delta_c = \frac{c}{\omega_{pi}\sqrt{\beta}}. \tag{12}$$

We then do the derivation of Sweet-Parker model again with $\Delta = \Delta_c$. From the mass conservation, we have

$$v_{in} = \frac{\Delta_c}{L} v_A = \frac{v_A}{L} \frac{c}{\omega_{pi}\sqrt{\beta}}. \tag{13}$$

and the reconnection time is

$$\tau \sim \frac{L}{\Delta_c v_{in}} = L^2 v_A \frac{\omega_{pi}\sqrt{\beta}}{c}. \tag{14}$$

The reconnection time $\tau \sim 5 \times 10^5$ seconds for the solar flare example [1], which is closer to the observation value but still roughly 500 times longer.

1.3 relativistic reconnection

In this section we investigate the relativistic reconnection. The relativistic reconnection happens when the magnetic energy before the field reconnects is much larger than the total enthalpy of the particles. Then the particles are relativistic before the reconnection. This can be quantified as relativistic magnetization parameter σ, [2] given as

$$\sigma = \frac{B_0^2}{4\pi mnc^2 \omega_n}. \tag{15}$$

The reconnection is relativistic if $\sigma > 1$. Note that $\omega_n = \gamma + p/(mnc^2)$ is the enthalpy per particle and p is the particle pressure and γ is the mean particle Lorentz factor.

Relativistic reconnection is very common in the astrophysical magnetized object, such as active galactic nuclei, pulsars, magnetars and gamma-ray bursts. And relativistic reconnection is often the mechanism for accelerating the particles to high energies. During the relativistic reconnection, the magnetized plasma goes toward the central current sheet with inflow velocity v_{in}. After that magnetic field line will change topologically. Then there is an outflow plasma in the original field line direction with the outflow velocity v_{out}. According to the momentum conservation, v_{out} is about the relativistic Alfven velocity v_A, written as

$$v_{out} = v_A = c\sqrt{\frac{\sigma}{1+\sigma}} \sim c \tag{16}$$

for $\sigma \gg 1$ in the relativistic regime.

We now consider the relativistic Sweet-Parker model [3]. We consider the 3D model with the current sheet in the xz plane. The width of the current sheet is 2Δ and the length of the current sheet is $2L$. Using the pressure balance across the sheet in y direction, we have

$$p = \frac{B_0^2}{8\pi}, \tag{17}$$

where p is the pressure at the middle of the current sheet and B_0 is the magnetic field outside the current sheet.

Next from the momentum equation, we have

$$\frac{\partial}{\partial x}\left(\omega \frac{v_{out}^2}{c^2}\gamma_{out}^2 - p\right) = -j_z B_y. \tag{18}$$

Here we can approximate $\partial/\partial x$ as $1/L$. Also we have

$$j_z = \frac{B_0}{4\pi\Delta} \tag{19}$$

from Ampere's law. Withe the flux conservation $B_y \sim \Delta B_0/L$ and the Ampere's law, we can write the momentum equation as

$$\omega \frac{v_{out}^2}{c^2}\gamma_{out}^2 = \frac{B_0^2}{8\pi}. \tag{20}$$

In the case of the inflow magnetic energy density much larger than the plasma rest energy density, we have the enthalpy as $\omega = 4p = B_0^2/2\pi$. By plugging the expression the ω into the above momentum equation, we can get that

$$\gamma_{out} \sim 1, \quad v_{out} \sim c. \tag{21}$$

So the outflow velocity is about speed of light, which is consistent with Eq. 16.

We then consider the energy conservation. The inflow energy is $v_{in}B_0^2/8\pi$ and the outflow energy is ωv_{out}. Note that the enethalpy can be written as $\omega = 4p = B_0^2/2\pi$. By using the energy balance,

$$v_{in}\frac{B_0^2}{8\pi}L = \omega v_{out}\Delta, \tag{22}$$

we will get

$$\frac{v_{in}}{v_{out}} \sim \frac{\Delta}{l}. \tag{23}$$

This relations between v_{in} and v_{out} also implies the flow is almost incompressible.

In the Maxwell–Faraday equation of steady state, we have $\nabla \times E = 0$. So the E_z is the same outside and within the current sheet. Within the current sheet, the Ohm's law implies that $E_z = \eta j_z$, where η is the resistivity. Outside the current sheet, the ideal MHD approximation gives that $E_z = (v_{in}/c)B_0$. So we have

$$E_z = \eta j_z = (v_{in}/c)B_0. \tag{24}$$

Then substitute the Ampere's law $j_z = B_0/(4\pi\Delta)$ and $\Delta = Lv_{in}/c$ into above equation, we will get

$$\frac{v_{in}^2}{c^2} = \frac{c\eta}{4\pi L}, \tag{25}$$

giving that

$$\frac{v_{in}}{c} = R_M^{-1/2}, \tag{26}$$

where $R_M = 4\pi L/(c\eta) \gg 1$. We can see that the inflow veleocity v_{in} is roughly non-relativistic. Also, this relativistic Sweet-Parker has the same scaling with R_M as the non-relativistic case.

Though the above discussion of the relativistic Sweet-Parker model doesn't include the kinetic effects, it still gives some feeling what the relativistic model will look like. The more detailed model including kinetic effect can be found in [4]. This work introduces the inertial ion length c/ω_{pi} into the analysis.

2 Laser-driven reconnection

In this section, we will talk about some laser experiments for magnetic reconnection. When the laser irradiates the plasma, the laser EM waves will accelerate the electrons to move forward. This will form an electric current, generating an azimuthal magnetic field. From the PIC simulation done by myself, this azimuthal magnetic field can be at most 20 percent of the laser magnetic field. So we can use self-generated azimuthal experiment to do some experiments for reconnection. Two laser adjacent are used to irradiate the plasma at the same time. Then two laser target will have the same azimuthal magnetic field. And in the middle of the two target, we will see that there's an opposite magnetic field line, which corresponding to the setup of the magnetic reconnection. In the following, I will review two laser experiments for the magnetic reconnection. The second one has a higher laser intensity than the first one, giving a higher azimuthal magnetic field.

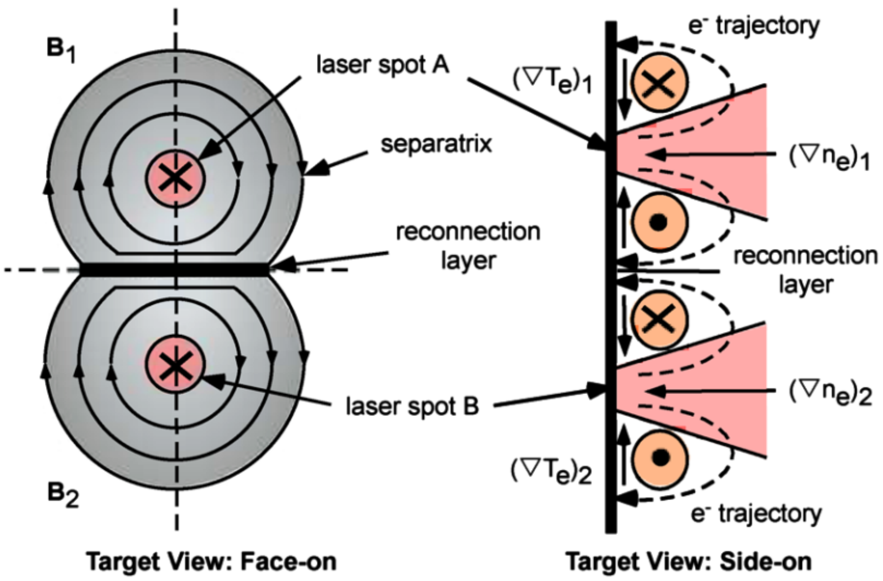

Figure 2: The geometry of the laser target and the magnetic field configuration. Courtesy of [5].

Nilson et al. (2006) [5] presents a laser experiment for magnetic reconnection at the Rutherford Appleton Laboratory, UK. Fig. 2 is the schematic of the experiment. Two lasers with intensity 1×10^{15} W cm^{-2} and wavelength $\lambda = 1.054$ micron are used to irradiate the aluminum or gold target foil. The two laser beams were aligned with different target distances. From the experiment results, the magnetic field of $0.7 \sim 1.3$ MG at the focal spot edges is observed.

For the aluminum plasma, the experiments show that the Alfven velocity is about $v_A = 1.35 \times 10^7$. We can see that the outflow velocity, or Alfven velocity, is much smaller than the speed of light. So this reconnection is non-relativistic. Other experiment parameters are $L = 100\ \mu m$, $T_e = 800\, eV$, $B = 1MG$. Using these parameters, we can get the Sweet-Parker reconnection rate as 4 ns, which is an approximate number since there are still some uncertainty in the length of the current sheet L. This experiment has confirmed that this is a magnetic reconnection but not a hydrodynamic collision without the magnetic fields. It can provide some studies on the microphysics, such as the heating mechanism and the particle acceleration mechanism.

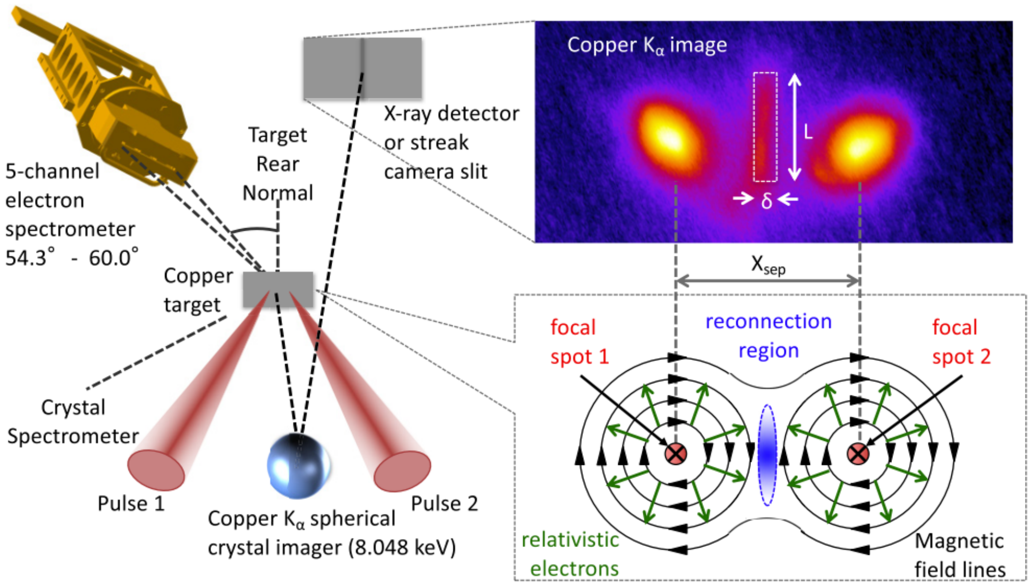

Figure 3: The experiment setup in [6].

Raymond et al. (2018) [6] presents laser experiments for relativistic reconnection. The experiments were performed both at the University of Michigan with $\lambda = 800\ \mu m$ and intensity of $2 \times 10^{19}\mathrm{Wcm}^{-2}$ normal incidence and the Laboratory of Laser Energetics (LLE) with $\lambda = 1.053\ \mu m$ and intensity of $1.2 \times 10^{18}\ \mathrm{Wcm}^{-2}$at 57.2^o incidence. The targets are both copper foil. The two laser beams are separated by a distance X_{sep}. The setup of the experiment is in Fig. 3.

According to the basics of the direct laser acceleration theory, the electron can be accelerated to relativistic energy when laser intensity is larger than 10^{18} W cm^{-2}. So it is evident that these experiment with intense lasers will generate the relativistic magnetic reconnection. It can also be seen in Fig. 4 that the electron energy goes to several MeV, which is highly relativistic. As for the strength of the magnetic field, this work doesn't present it. The experiments done on the same laser in Michigan [7] shows that the laser of intensity 4×10^{19} Wcm^{-2} can generate the magnetic fields of order 100 MG.

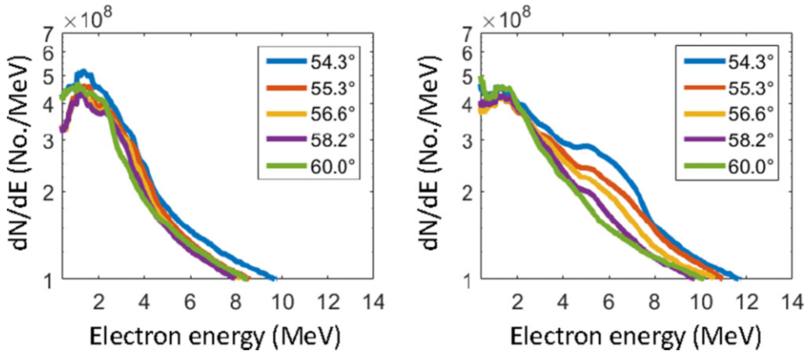

Figure 4: Electron energy spectrum from the experiment in LLE. [6]

This work [6] also provides a 3D PIC simulation result. The simulation setup is similar the real experiment. The simulation results show that the azimuthal magnetic field near the reconnection layer is about 20 MG. And the calculated magnetization parameter σ is larger than 10, which satisfy the criterion for the relativistic reconnection. However, the comparison of relativistic Sweet-Parker model with the experiments is not yet clear.

To model the relativistic reconnection more accurately, the quantum electrodynamics (QED) effect must be considered. The laser-generated plasma is now very close to the astrophysical conditions like the AGN or gamma-ray bursts. At such high energy system, the QED effect such as radiation reaction can be significant. The more detailed analysis of the magnetic reconnection with QED effect can be found in Bulanov (2016) [8].

3 Conclusion

In this report, we reviewed the basics of the Sweet-Parker model for the magnetic reconnection. It shows that the inflow velocity is equal to the outflow velocity multiplied by $R_M^{-1/2}$, where R_M is the magnetic Reynolds number. Then we discussed the non-MHD model that the ion-acoustic instability is taken into account. And the non-MHD model gives that the the inflow velocity is proportional to the ion inertial length c/ω_{pi}. Next the derivation of the relativistic Sweet-Parker model is given. The outflow velocity is about speed of light and the inflow velocity is much smaller than the speed of light. The inflow velocity in relativistic case has the scaling of $R_M^{-1/2}$, which is the same as the non-relativstic case.

As for the experiment, we reviewed two laser experiments for the reconnection. The reconnection can be produced in laboratory by using two laser focuses on adjacent spots, giving an opposite magnetic field in the middle of the two spots. Nilson et al. (2006) [5] presents a laser experiment for the non-relativistic reconnection. Raymond et al. (2018) [6] presents a relativistic reconnection experiment using lasers. To more accurately model the relativistic reconnection, the quantum electrodynamics effect must be taken into account.

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