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Sawtooth Disruption

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ABSTRACT

A rubber band has many similar characteristics to magnetic fields. Although imprecise it can be used to help picture physics in magnetic fields. One particular example is the stretching of a rubber band. The more you stretch a rubber band by inserting objects into it the more energy it stores. This essentially will lead to the rubber band breaking. Unfortunately, if this individual is studying plasma physics, they will be able to relate this to the phenomena of magnetic reconnection. In a similar process to the rubber band, a set of magnetic fields can break simultaneously. Since $\nabla \cdot \mathbf{B} = 0$, magnetic fields cannot have loose ends, therefore, they will undergo a reconnection event. In the process of reconnection, the magnetic fields release an enormous amount of energy. This is a very common process when it comes to plasmas. It can be observed in solar flares, active galactic nuclei jets, and laboratory experiments on Earth such as fusion. In the case of fusion, magnetic reconnection has made a significant impact. In plasma confinement devices such as tokamaks, perfect confinement is not possible due to disruptive events caused by magnetic reconnection. In this paper a discussion regarding sawtooth disruptions associated with magnetic reconnection in fusion devices will be provided. In addition to this a discussion regarding the historical development and recent progress will be provided. To further the understanding of the scientific community, suggestions for future experiments will be discussed.

Keywords: Reconnection, Resistive MHD, Sawtooth, Kink Instability

1. INTRODUCTION

Sawtooth oscillations are a common observations in laboratory fusion devices. This phenomena was observed and studied back in 1974 by Goeler *et al* in the spherical torus (ST) tokamak. A continuous soft-x-ray emission from the hot central core of the ST tokamak was analyzed in which traces exhibited a “sawtoothlike” oscillation, (14).

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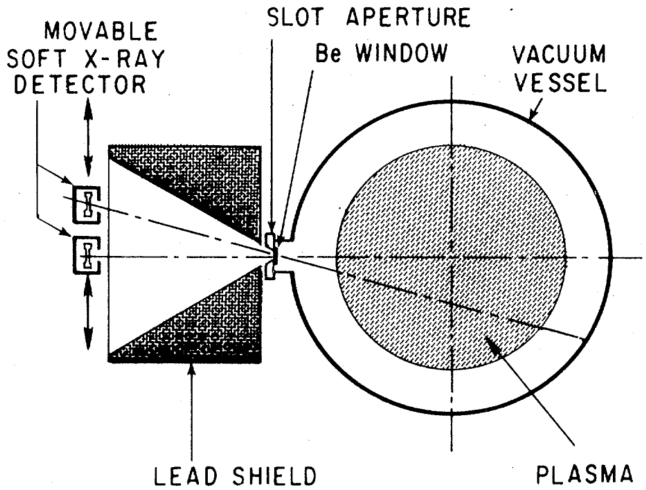
Similar sawtooth oscillations seem to be present also in the T-4 tokamak and had also been seen in ATC tokamak. In this preliminary study, Goeler *et al*, showed that these observations have the features of internal disruptions preceded by $m = 1$ oscillations.

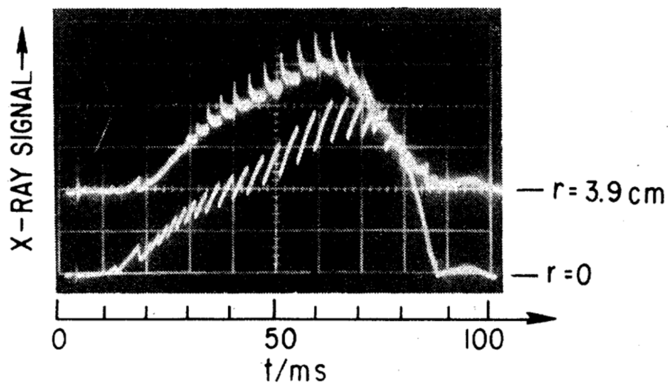
Since 1974, sawtooth events are now a common feature seen in every tokamak. Different experiments are conducted to either suppress the sawtooth oscillation or make it in such a way that it benefits the system overall. Before discussing possible solutions or modifications to tokamak experiments it is of benefit to discuss the typical sawtooth cycle. The typical sawtooth event consists of a period in which the plasma density and temperature increases, followed by the growth of a helical magnetic perturbation, which is shortly followed by a rapid collapse of the central pressure. The sawtooth event is a large-scale instability that affects significantly the volume of the plasma. The sawtooth crash leads to an event where hot electrons transport rapidly across flux surfaces to a cooler region of the plasma, thus, flattening the temperature profile. In experimental observations, at each sawtooth crash the central soft x-ray emission shows a rapid drop in the core temperature followed by a heating of the edge plasma.

The event leading up to the disruptions is caused by growing sinusoidal $m = n = 1$, oscillations, (14). This is associated with the existence of a $q = 1$ surface in the plasma. In the next section this will be discussed in further detail. After the first instability was observed the first model that explains the fast magnetic reconnection was proposed by Kadomtsev, (7). The Kadomtsev model successfully explains some of the features of the sawtooth crash, but after further analysis and experiments in bigger tokamaks, it was found that the Kadomtsev model does not paint the full picture. In Sec. 2, a description of the Kadomtsev model will be provided. An alternative explanation provided by Wesson was brought forth in 1989 after sawtooth oscillations were observed on JET, (16). This other model was developed due to present models not being consistent with experimental observations. Wesson's model offers a description based on non-linear evolution of quasi-interchange mode, which does not involve much reconnection. Due to limited time, the Wesson model will not be discussed in this project. This instead could be a project interest for the next 218B class.

Now that we know of the growing instability leading up to the sawtooth crash, we can further dive into the underlying physics behind this. In the rest of this paper, I will provide a quick explanation on magnetic reconnection based on a back-of-envelope derivation of the Sweet-Parker model, then I'll discuss the Kadomtsev model in further detail, and finally I'll discuss the tokamak instability known as the kink instability associated with the $m = n = 1$ observed in the events leading up to the sawtooth crash. It should be mentioned that it is unclear what the instability is. It may be the internal kink mode, resistive internal kink mode, or the $m = 1$ tearing mode. In this paper I'll discuss the first, but a future project idea can be to explore the $m = 1$ tearing mode. Here is a brief outline of the paper and the topics that will be discussed. In Sec. 2, a detailed discussion of magnetohydrodynamics (MHD) models



(a)



(b)

Figure 1: Experimental arrangment of x-ray detectors. The x-ray traces exhibit internal disruptions. Image is taken from (14).

that attempt to describe the sawtooth oscillation will be provided, later in Sec. 3, a brief overview of observations and recent developments made to control sawtooth events will be provided. At the end of the paper in Sec. 4, I will provide a summary of the topics discussed and mention ideas that might be explored in order to further the understanding of sawtooth events in laboratory experiments.

2. SAWTOOTH PHYSICS

2.1. *Sweet-Parker Reconnection*

The first theoretical framework of magnetic reconnection is known as the Sweet-Parker (S-P) model. In magnetic reconnection the breaking of the freezing-in of magnetic fields is now allowed,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}) + \mu \nabla^2 \mathbf{B}. \tag{1}$$

The S-P model describes the time-independent magnetic reconnection in the resistive MHD framework when the reconnecting magnetic fields are in opposite direction and effects due to viscosity and compressibility are not important. A back-of-envelope derivation of the S-P reconnection rate can be derived by matching in-flow to out-flow and using mass, momentum, and energy conservation. From class the reconnection rate is given as,

$$\frac{1}{\tau_R} \propto S^{-1/2}, \tag{2}$$

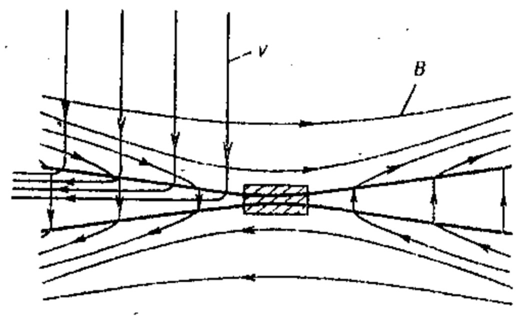


Figure 2: Plasma expulsion along the layer during reconnection. Image is taken from (8).

where S is the Lundquist number, which is a dimensionless ratio that compares the timescale of an Alfven wave and the resistive diffusion,

$$S = \frac{L v_A}{\eta}. \tag{3}$$

For dynamics of interest the typical Lundquist number is $\sim 10^3 - 10^8$. The simple model of S-P reconnection shown describes a lot of the main features of the phenomenon quite satisfactorily. Nevertheless, experimental data of sawtooth oscillations indicate a variety of possible forms of reconnection such as partial reconnection. Yet, to fully understand the behaviour of the sawtooth process, it is crucial to first understand the S-P model.

2.2. *Kadomtsev Model*

Two sets of models have been proposed to explain the phenomena of Sawtooth oscillations. The first model proposed was given by Kadomtsev back in 1975 (7), the other model that offers an explanation is the Wesson model, which describes sawtooth relaxation based on the non-linear evolution of the quasi-interchange mode, (15). The Wesson model does not involve much reconnection, thus, it lacks many of the S-P scaling. The most commonly used is the Kadomtsev model, which this paper will focus on. The Kadomtsev model came forth as a means to explain the fast magnetic reconnection observed in the sawtooth collapse.

The derivation of the Kadomtsev collapse time can be derived as follows: Magnetic fields on a $q = 1$ surface define a helix. Consider the magnetic field lines to wind around a torus with $d\theta/d\phi = 1$. A helical flux arises when the magnetic field lines intersect a sheet where $q \neq 1$. The unit normal to the sheet has the following,

$$\hat{\theta} = \frac{1}{\sqrt{1 + r^2/R^2}}, \tag{4}$$

$$\hat{\phi} = -\frac{r/R}{\sqrt{1 + r^2/R^2}}. \tag{5}$$

The magnetic field is given as

$$B = B_\theta - (r/R)B_\phi \tag{6}$$

$$= B_\theta(1-q), \tag{7}$$

here q represents the safety factor. The helical flux, defined as $d\psi/dr = B$, between the axis and the $q = 1$ surface reconnects with an equal and opposite flux outside the $q = 1$ surface. The flux is continuously reconnected and the reconnected flux forms an island that grows and takes the place of the original nested flux surfaces. The reconnection can be imagined to take place in a layer of thickness δ as shown in Fig. 3. By using Ampere's law and noting that there is a current sheet in the layer due to the electric field the velocity v_1 can be derived to be,

$$v_1 \sim \frac{\eta}{\delta}. \tag{8}$$

The movement of the core gives a pressure which causes flow along and out of the layer with velocity v_2,

$$\rho v_2^2 \sim B^2. \tag{9}$$

Applying the relations to $v_1 r_1 \sim v_2 \delta$ we can arrive to the following,

$$\delta \sim \left(\frac{\tau_A}{\tau_R}\right)^{1/2} r_1, \tag{10}$$

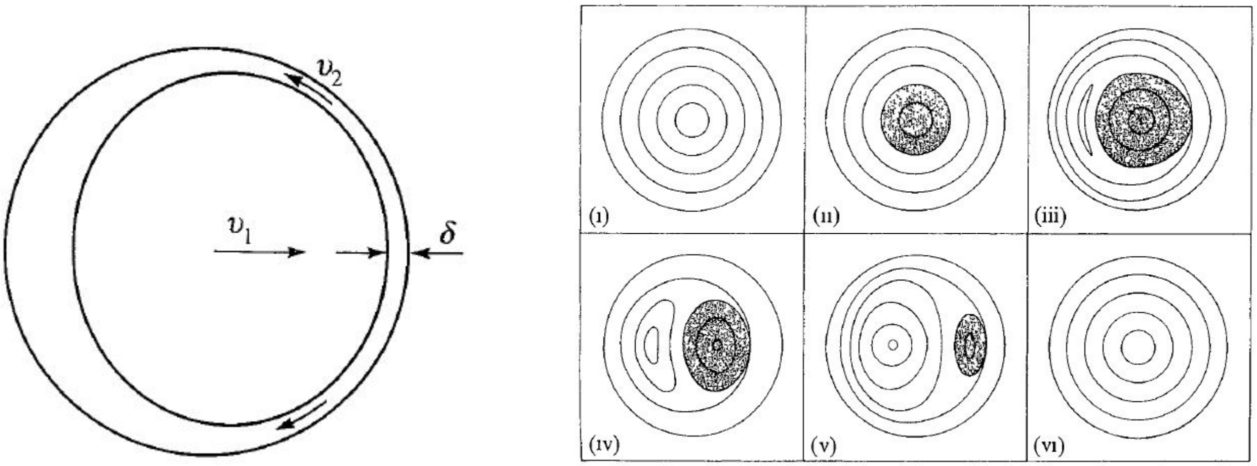
where τ_R and τ_A represent the resistive diffusion time and the Alvenic time. Furthermore, the time of the collapse is give by the time for the core to move across the radius r_1,

$$\tau_K \sim \frac{r_1}{v_1}, \tag{11}$$

which can be rewritten as,

$$\tau_K \sim (\tau_R\ \tau_A)^{1/2}. \tag{12}$$

Here $\tau_R \gg \tau_A$, thus, the predicted collapse time agrees with observed experimental values in small tokamak devices. Numerical simulations have confirmed the Kadomtsev's model to be a solution to the resistive MHD equations, but experiments and the model do not match exactly.



(a) (b)

Figure 3: **a)** Reconnection takes place in narrow layer of thickness δ, the central core moves into the layer with v_1 and plasma leaves layer with v_2. **b)** Development of the magnetic field structure during the sawtooth instability according to Kadomtsev's model. The $m = 1$ instability displaces the $q < 1$ region and restores q_0 to a value > 1. Image is taken from (17).

From eq. 12, the collapse time is proportional to $r_1^{3/2}$, therefore, there should be an increase for larger fusion devices. For example, the JET collapse times are predicted to be ~ 10 ms, but instead its observed to be 100 μs. Characteristic times for JET sawteeth are given in (15). It is clear from this that the Kadomtsev model is not taking into account other factors. One other factor is that in high temperature plasmas the electrical impedance of the reconnection layer is not resistive and that the reconnection is collisionless. The rate can therefore be determined by the electron inertia. By using Ohm's law and the electron collision time a much more general solution can be derived,

$$\tau \sim \frac{\tau_A}{\left(\tau_A/\tau_R + (c/r_1\omega_p)^2(1 + (\beta_e/(m_e/m_1))^{1/2})\right)^{1/2}}. \tag{13}$$

The condition for the electron viscosity to dominate the electron inertia is given by $\beta_e > m_e/m_1$, where $\beta \propto nT_e/B_\phi^2$.

2.3. *Kink Instability*

From observation, the sawtooth crash is usually accompanied by an $m = n = 1$ kink displacement, where m and n are poloidal and toroidal mode numbers, respectively. Here I'll derive an expression for the energy change resulting from a displacement ξ of the plasma, δW. The strongest ideal MHD instability is the kink mode. To derive an expression for δW up to leading order terms by considering a cylinder. The calculation can be carried out using (r, θ, ϕ) where $(R\ \phi)$ corresponds to the z coordinate of a cylinder. From the energy principle we have the following expression,

$$\delta W = \frac{1}{2}\int_{\text{plasma}}\left(\gamma p_0(\nabla\cdot\xi)^2 + (\xi\cdot\nabla p_0)\nabla\cdot\xi + \frac{B_1^2}{\mu_0} - \mathbf{j}_0\cdot(\mathbf{B_1}\times\xi)\right)d\tau + \int_{\text{vacuum}}(B_v^2/2\mu_0)d\tau. \tag{14}$$

Taking the following approximations, $\partial/\partial r \sim (1/r)\partial/\partial\theta \gg (1/R)\partial/\partial\phi$, the minimizing perturbation has $\nabla\cdot\xi = 0$, $\xi_r \sim \xi_\theta \gg \xi_\phi$ and $B_{r1} \sim B_{\theta 1} \gg B_{\phi 1}$, which gives

$$\delta W = \pi R\int_0^a\left(\frac{B_1^2}{\mu_0} - j_{z0}(B_{r1}\xi_\theta - B_{\theta 1}\xi_1)\right)r\ d\theta\ dr + \pi R\int_a^b \frac{B_v^2}{\mu_0}r\ d\theta\ dr, \tag{15}$$

where a and b are the radius of the plasma and conducting wall, respectively. Consider now an MHD oscillation of the form $\exp(im\theta - in\phi)$. From $\nabla\cdot\xi$ we have the following,

$$\xi_\theta = -\frac{i}{m}\frac{d}{dr}(r\xi_r), \tag{16}$$

then using $\mathbf{B}_1 = \nabla\times(\xi\times\mathbf{B}_0)$, the magnetic perturbations are

$$B_{r1} = -\frac{imB_\phi}{R}\left(\frac{n}{m} - \frac{1}{q}\right)\xi_r, \tag{17}$$

$$B_{\theta 1} = \frac{B_\phi}{R}\frac{d}{dr}\left[\left(\frac{n}{m} - \frac{1}{q}\right)r\xi_r\right]. \tag{18}$$

The safety factor q mentioned earlier is defined as

$$q = \frac{rB_\phi}{RB_\theta}. \tag{19}$$

From here all these equations can be applied to eq. 14 and then we have the following expression for the potential energy of the perturbations after cancellations and integration by parts,

$$\delta W = \frac{\pi^2 B_\phi^2}{\mu_0 R}\int\left[\left(r\frac{d\xi}{dr}\right)^2 + (m^2-1)\xi^2\right]\left(\frac{n}{m} - \frac{1}{q}\right)^2 r\ dr + O(\xi^2). \tag{20}$$

Now that there is an expression for the general kink instability given by eq. 20 a discussion regarding the internal kink can now be given. There is a relationship between the azimuthal mode number and the toroidal mode number given by $m = nq$. The resonant surface for this mode is $q = 1$, thus this is when the stability is determined by higher order ξ^2 terms. Therefore, a condition for stability requires the minimum value for q to be greater than unity.

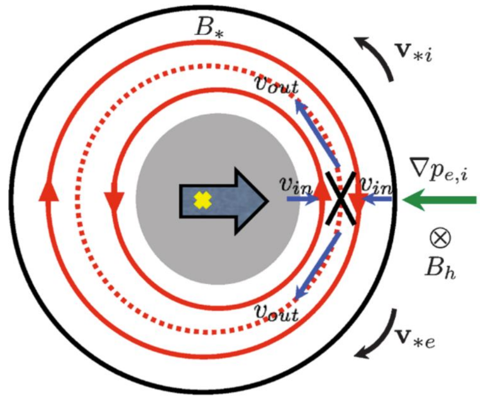


Figure 4: Image of $m = n = 1$ reconnection plane. Plasma inflows and outflows are denoted as v_{in} and v_{out}, respectively. Reconnection site is indicated as a black X. Core moves from initial position (yellow X). Image is taken from (1).

The ideal internal kink displacement takes the form of a tilt and shift of the core plasma. This instability has mode number $m = 1$ and it affects the plasma core when $q \leq 1$ and its driven by the pressure gradient, (13).

To conclude this, it should be noted that the sawtooth stability is not determined solely by the 1/1 internal kink mode. The dynamics can be affected by other factors such as energetic particles, sheared flows, pressure anisotropy, diamagnetic effects, nonlinear reconnection physics, and local effects in the inertial layer around the $q = 1$ surface, (3). In the next section, I'll provide a brief review of recent development concerning the stability of the internal kink and control methods that are planned for future laboratory experiments.

3. REVIEW OF CURRENT KNOWLEDGE

As discussed in the previous section, the sawtooth phenomena is not only determined by the 1/1 internal kink mode. One dynamic that plays a role is energetic particles. In tokamaks, the energetic particles are due to ion cyclotron resonance heating (ICRH), neutral beam injection (NBI), or α particles from fusion reactions, (2). According to Porcelli *et al*, it is found that sawteeth can be stabilized by α particles in ITER for longer periods, (10). Sawteeth with longer periods are known as 'monster' sawteeth. From experimental observation it is known that energetic particles can stabilize the plasma. The theoretical model that explains this phenomena is the Fishbone instability, which is the interaction between injected particles and $m = n = 1$ MHD perturbation. More information regarding the fishbone instability can be found here, (18).

In addition of energetic particle, it should be noted that NBI actually leads to plasma rotation and flow shear, (5). The toroidal rotation of the plasma can approach the ion thermal speed. Here the centrifugal effects caused by the toroidal rotation can affect the kink mode stability. In (9), it is found that the toroidal sheared flows with speed similar to the sound speed stabilize the sawteeth in tokamaks. The $n = 1$ resistive tearing mode is completely stabilized by the flow. Thus, stabilization is not only due to energetic particles, but also plasma rotation.

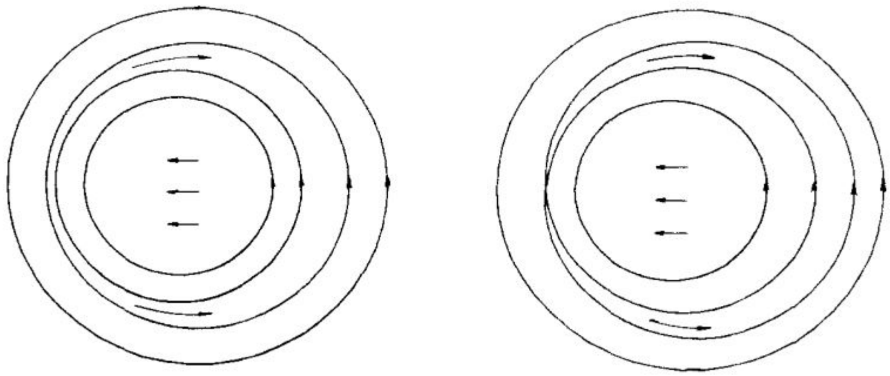
So far only the ideal MHD and kinetic effects have been discussed to trigger the sawtooth crash. It should be mentioned though that also non-ideal effects such as electrical resistivity, electron compressibility, diamagnetic effects and other neoclassical effects can also affect the stability of the sawtooth crash. However, experiments that were later conducted in larger tokamaks showed that the Kadomtsev model was insufficient to explain the phenomena of sawtooth crashes. Other models were later proposed. Crash trigger models such as resistive two-fluid MHD, collisionless kinetic effects, magnetic stochastization, and many more, (20; 6).

To conclude this section, I'll discuss methods on how sawtooth events may be controlled. Sawtooth control can come in the form of plasma shaping to heating or current driven system to alter the sawtooth period. The two approaches are: i) stabilize i.e., eliminate or delay the sawtooth crash; ii) destabilize i.e., decrease the sawtooth period and reduce as much as possible the triggering of other MHD instabilities. Some methods which were discussed earlier that can be used as controls are NBI, electron cyclotron resonance heating and electron cyclotron current drive (ECCD). Note that ECCD is something that will be included as a sawtooth control in ITER, (11). Examples of current drive schemes that can be applied to plasmas comes from ECRH, which can increase the likelihood of a sawtooth crash. This is also included in the design of the sawtooth control system for ITER. Other methods that affect the sawtooth behaviour are lower hybrid current drive and mode conversion current drive, (12; 19).

4. CONCLUSION

The phenomena of the sawtooth crash in tokamak devices is very complex. The good thing is that there are many ideas that attempt to explain the phenomena and there are many experiments going on that are constantly testing how it can be avoided or controlled so that it benefits fusion devices. For example, sawteeth is expected to be of use in ITER by reversing the on-axis accumulation of higher-Z impurities that can cause degradation of energy confinement due to impurity radiation, (3). Therefore, the plan on ITER is to destabilize the internal kink mode to give frequent small sawtooth crashes.

The understanding of the sawtooth event has been in development for a long time now. As discussed early in Sec. 2, the idea of the sawtooth event can be traced back to as early as the Sweet-Parker model for magnetic reconnection. Shortly after we have the model that is always referred to in the literature, the Kadomtsev model. The impact of the Kadomtsev model is remarkable, since it was able to explain many of the phenomena that occurs during the sawtooth crash. Although the model worked for many of the small tokamak devices it failed to reproduce the collapse time for larger tokamaks. The inconsistency of the model therefore led to closer inspection of the instability that usually accompanied the sawtooth crash. As discussed, this instability is the internal kink instability. Regardless, it should be noted that the sawtooth stability is not solely determined by the 1/1 internal kink mode. Other dynamics can affect the stability.



(a) (b)

Figure 5: Perturbed flux surfaces. **a)** Ideal kink mode. **b)** The $m = 1$ tearing mode. Arrows indicate the direction of plasma flow. In both cases pressure balance requires a large return flow near the $q = 1$ surface. Image is taken from (4).

As discussed in Sec. 3, a dynamic that affects the stability of the of the plasma comes from energetic particles such as ICRH, NBI, and α particles. Due to experiments, there has been considerable progress in theoretical understanding of the sawtooth process. Nowadays, some of these instabilities are used to control the sawtooth event and make use of the features as described in the first paragraph of this section. Methods that can stabilize or destabilize the plasma are NBI, electron cyclotron resonance heating and ECCD. Yet, there is still a lot of uncertainty. For example, it is unclear how some of these methods which will be implemented in ITER behave. Some of these questions are, how long will the sawtooth period be on ITER? Will the length of the period trigger any other instabilities?

To answer these questions, further experiments and numerical simulations need to be performed to come up with a better sawtooth model that can explain the uncertainties. As mentioned earlier, there are other instabilities other than the kink instability that play a role, which also needs to be accounted for. One of these is the tearing instability. There are other models as well that should be discussed in further detail such as the Wesson Model. These are all potential topics for projects for the next 218B class. Even though the physics explanation for the sawtooth oscillation remains incomplete, there are still well established ideas and methods that can at least allow us to continue to ask questions so that one day a complete model that explains the phenomena can be achieved.

REFERENCES

[1] M. T. Beidler and P. A. Cassak. Model for incomplete reconnection in sawtooth crashes. *Phys. Rev. Lett.*, 107:255002, Dec 2011.

[2] D. J. Campbell, D. F. H. Start, J. A. Wesson, D. V. Bartlett, V. P. Bhatnagar, M. Bures, J. G. Cordey, G. A. Cottrell, P. A. Dupperex, A. W. Edwards, C. D. Challis, C. Gormezano, C. W. Gowers, R. S. Granetz, J. H. Hammen, T. Hellsten, J. Jacquinot, E. Lazzaro, P. J. Lomas, N. Lopes Cardozo, P. Mantica, J. A. Snipes, D. Stork, P. E. Stott, P. R. Thomas, E. Thompson, K. Thomsen, and G. Tonetti. Stabilization of sawteeth with additional heating in the jet tokamak. *Phys. Rev. Lett.*, 60:2148–2151, May 1988.

[3] I T Chapman. Controlling sawtooth oscillations in tokamak plasmas. *Plasma Physics and Controlled Fusion*, 53(1):013001, nov 2010.

[4] J. F. Drake. Kinetic theory of m=1 internal instabilities. *The Physics of Fluids*, 21(10):1777–1789, 1978.

[5] J P Graves, R J Hastie, and K I Hopcraft. The effects of sheared toroidal plasma rotation on the internal kink mode in the banana regime. *Plasma Physics and Controlled Fusion*, 42(10):1049–1066, oct 2000.

[6] S. C. Jardin, I. Krebs, and N. Ferraro. A new explanation of the sawtooth phenomena in tokamaks. *Physics of Plasmas*, 27(3):032509, 2020.

[7] B B Kadomtsev. Tokamaks and dimensional analysis. *Sov. J. Plasma Phys. (Engl. Transl.); (United States)*.

[8] Boris Borisovich Kadomtsev. Tokamak plasma : a complex physical system. 1992.

[9] Robert G. Kleva and Parvez N. Guzdar. Stabilization of sawteeth in tokamaks with toroidal flows. *Physics of Plasmas*, 9(7):3013–3018, 2002.

[10] F Porcelli, D Boucher, and M N Rosenbluth. Model for the sawtooth period and amplitude. *Plasma Physics and Controlled Fusion*, 38(12):2163–2186, dec 1996.

[11] M Shimada, D.J Campbell, V Mukhovatov, M Fujiwara, N Kirneva, K Lackner, M Nagami, V.D Pustovitov, N Uckan, J Wesley, N Asakura, A.E Costley, A.J.H Donné, E.J Doyle, A Fasoli, C Gormezano, Y Gribov, O Gruber, T.C Hender, W Houlberg, S Ide, Y Kamada, A Leonard, B Lipschultz, A Loarte, K Miyamoto, V Mukhovatov, T.H Osborne, A Polevoi, and A.C.C Sips. Chapter 1: Overview and summary. *Nuclear Fusion*, 47(6):S1–S17, jun 2007.

[12] F.X Soldner, F Leuterer, R Bartiromo, S Bernabei, R Buchse, O Gehre, R.W Harvey, M Kornherr, K McCormick, H.D Murmann, and G.V Pereverzev. Profile control with lower hybrid waves on ASDEX. *Nuclear Fusion*, 34(7):985–1016, jul 1994.

[13] G Vlad and A Bondeson. Numerical simulations of sawteeth in tokamaks.

[14] S. von Goeler, W. Stodiek, and N. Sauthoff. Studies of internal disruptions and $m = 1$ oscillations in tokamak discharges with soft—x-ray tecniques. *Phys. Rev. Lett.*, 33:1201–1203, Nov 1974.

[15] J A Wesson. Sawtooth oscillations. *Plasma Physics and Controlled Fusion*, 28(1A):243–248, jan 1986.

[16] J.A. Wesson, R.D. Gill, M. Hugon, F.C. Schüller, J.A. Snipes, D.J. Ward, D.V. Bartlett, D.J. Campbell, P.A. Duperrex, A.W. Edwards, R.S. Granetz, N.A.O. Gottardi, T.C. Hender, E. Lazzaro, P.J. Lomas, N. Lopes Cardozo, K.F. Mast, M.F.F. Nave, N.A. Salmon, P. Smeulders, P.R. Thomas, B.J.D. Tubbing, M.F. Turner, and A. Weller. Disruptions in JET. *Nuclear Fusion*, 29(4):641–666, apr 1989.

[17] John Wesson. *Tokamaks*. 2004.

[18] R. B. White, P. H. Rutherford, P. Colestock, and M. N. Bussac. Sawtooth stabilization by energetic trapped particles. *Phys. Rev. Lett.*, 60:2038–2041, May 1988.

[19] S. J. Wukitch, Y. Lin, A. Parisot, J. C. Wright, P. T. Bonoli, M. Porkolab, N. Basse, E. Edlund, A. Hubbard, L. Lin, A. Lynn, E. Marmar, D. Mossessian, P. Phillips, and G. Schilling. Ion cyclotron range of frequency mode conversion physics in alcator c-mod:experimental measurements and modeling. *Physics of Plasmas*, 12(5):056104, 2005.

[20] L. Zakharov and B. Rogers. Two-fluid magnetohydrodynamic description of the internal kink mode in tokamaks. *Physics of Fluids B: Plasma Physics*, 4(10):3285–3301, 1992.