Analysis of shear flow stabilization conditions for Z-pinch configuration plasmas in theory, simulation, and experiment

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1 Abstract

Z-pinch configuration plasmas offer an attractive alternative for fusion reactor design when compared to other confinement methods, primarily due to the high β ratio they can achieve for confining the plasma. This allows for relative miniaturization of the device compared to other fusion devices; however, z pinches are also susceptible to instabilities not only from MHD dynamics but also drift modes on the ion gyroradius scale. It has been shown recently that an axially sheared plasma flow can help maintain equilibrium by lowering the growth rate of these instabilities. The goal of this report is to analyze the theory, simulations, and experiments related to shear flow stabilized z-pinches in order to boil down the shear conditions needed to maintain stability in a generalized z-pinch.

2 Introduction

The problem of achieving viable nuclear fusion is being actively research from many different angles. One of the reactor configurations which has been around since the early days of plasma physics research, and is experiencing a resurgence, is the Z-Pinch.

Z-Pinches are relatively simple confinement configuration of a cylindrical plasma with an axial current. The plasma is compressed, as if a bunch of wires, by the azimuthal magnetic field produced by the primary current, helping confine the plasma. Some advantages of Z-pinch which allow for smaller machines is that they do not need external magnetic field coils for stability and the average β ratio of thermal pressure to magnetic pressure is of order one. The clearest downside to Z-pinches are rapidly growing instability modes, primarily the sausage m=0 and the kink m=1 modes; however, various schemes have been developed to mitigate these instabilities enough so that Z-pinches have resurged to a position of prominence contending for the coveted net gain equilibrium state.

For example, one can embed an axial magnetic field in the plasma; however, this changes the force balance, reduces the confinement effiency by reducing the β ratio, and increases heat lost via conduction on electrodes. Pulsed power implosion drivers can produce transient Z-pinch plasma, but more research is needed to mitigate the subsequent growth of instabilities on the timescale of acoustic propegation across the pinch radius. Conducting walls close to the surface of the plasma can stabilize the z pinch via their surface charges yet it has been found the walls must ideally be within radius $\frac{r_{wall}}{a} < 1.2$ where a is the pinch radius. As conditions scale up one can imagine the problems of wall stabilization. In an attempt to outpace instabilties, the axial B and liner implosion concepts have been combined, dubbed magneto-inertial fusion (MIF) and magnetized liner inertial fusion (magLIF), leading to noteable results.[2] The ideal reactor approach would arguably be an equilibrium state though, so dynamic or "fast" Z-pinches will not be the focus.

Enter the sheared-flow stabilized Z-pinch (SFS Z-Pinch), which utilizes $\frac{dv_z}{dr}$, radial shear of the axial plasma velocity, to enhance stability. The shear flow conveniently doesn't affect the force balance used to establish equilibrium & leads to a 'quiescent period' in which plasma fluctuations are low and the flux tube is more centered and uniform in the assembly region.[1]

The following sections address theoretical scaling relationships showing how increasing the current 'pinches' the plasma column, stability properties under a sheared flow regime, findings from simulation and experimental studies, optimizations such as nonuniform flow shear and wall design, and open questions regarding the fundamental question of stability.

3 Theory

The equilibrium utilized is described by the radial force balance equation,

$$\frac{B_\theta}{\mu_o r}\frac{d(rB_\theta)}{dr} = -\frac{dp}{dr} \tag{1}$$

or more precisely in a charge neutral two-fluid treatment,

$$\frac{B_\theta}{\mu_o r}\frac{d(rB_\theta)}{dr} = -\frac{d}{dr}(n_i k_B T_i + n_e k_B T_e) \tag{2}$$

where $n_{i,e}, T_{i,e}$ are the ion and electron density and temperature, respectively, μ_o is the permeability of free space, and B_θ is the azimuthal magnetic field. Note that no axial magnetic field is used, not a 'screw pinch', meaning no need for external field coils.

The average β of the Z-pinch is defined

$$\langle\beta\rangle = \frac{\langle p\rangle}{B_\theta^2(r_w)/2\mu_o} \tag{3}$$

Notice if plasma pressure vanishes at distant wall r_w, where the axial current returns to ground, then $\langle\beta\rangle = 1$ as noted previously.

Integrating Eq. (2) over plasma volume produces the Bennett relation

$$(1+Z)N_i k_B\langle T\rangle = \frac{\mu_o I^2}{8\pi} \tag{4}$$

with ionization rate Z and axial current I, $\langle T\rangle$ average temperature, assuming $T = T_e = T_i$, and N_i is ion density defined over a cross section of the pinch volume. The Bennett relation remains valid for equlibrium density and magnetic field profiles, and leads to a number of scaling relations,

$$\frac{T_2}{T_1} = \left(\frac{I_2}{I_1}\right)^2\frac{N_1}{N_2} \tag{5}$$

If plasma compression occurs adiabatically then the following approximation is valid

$$0 = \frac{d}{dt}\frac{p}{n^\gamma} \tag{6}$$

with adiabatic index γ.

Utilizing a 'sharp pinch' model with equilirium conditions of uniform density and temoerature, $n_i = n$, $n_e = Zn$, and $T = T_e = T_i$, inside the pinch radius a, beyond which density vanishes, leads to density scaling

$$\frac{n_2}{n_1} = \left(\frac{I_2}{I_1}\right)^{\frac{2}{\gamma-1}}\left(\frac{N_1}{N_2}\right)^{\frac{1}{\gamma-1}} \tag{7}$$

and pinch radius scaling

$$\frac{a_2}{a_1} = \left(\frac{I_1}{I_2}\right)^{\frac{1}{\gamma-1}}\left(\frac{N_2}{N_1}\right)^{\frac{\gamma}{2(\gamma-1)}} \tag{8}$$

pressure scaling

$$\frac{p_2}{p_1} = \frac{n_2}{n_1}\frac{T_2}{T_1} \tag{9}$$

and finally B scaling

$$\frac{B_2}{B_1} = \frac{a_1}{a_2}\frac{I_2}{I_1} \tag{10}$$

Reaching a high-energy density or thermonuclear regime then relies primarily on increasing the current which then decreases pinch radius and increases density and temperature within the region. [4]

An exciting result; however, the model assumes equilibrium state so the interesting question then becomes how to enhance and maintain stability of the pinch.

4 Stability

It's worth addressing the background of static z-pinch stability which goes back to Kadomtsev who showed the equilibrium is unstable to MHD modes.[4] The azimuthal, axisymmetric sausage mode can be stabilized under the pressure gradient condition

$$-\frac{d(ln[p])}{d(ln[r])} \leq \frac{4\gamma}{2+\gamma\beta} \tag{11}$$

where β is measured locally. Taking $\gamma = \frac{5}{3}$ we find a parametric form for the equilibrium pressure,

$$r = a\frac{(\frac{4}{5}+\beta)^{1/4}}{\beta^{3/4}} \tag{12}$$

$$p = p_o\left(\frac{\beta}{\frac{4}{5}+\beta}\right)^{5/2} \tag{13}$$

where p_o is measured on axis.

Furthermore, kink modes characterized by $m = 1$ may be stabilized by yet another pressure gradient condition,

$$-\frac{d(ln[p])}{d(ln[r])} \leq \frac{m^2}{\beta} \tag{14}$$

Note that higher order modes $m \geq 2$ are stable if the kink mode is stable so the primary concern is with the sausage and kink modes.

Unfortunately, no realizable pressure profile exists to satisfy the $m = 1$ condition without a rigid conductor placed along the primary axis of the configuration, which motivates solutions other than profile optimization as sugested in the introduction.

The report is focused on SFS Z-pinch: recall wall stabilization requires a close wall, and that stabilization by axial B, according to the Kruskal-Shafranov limit, limits the axial current driving the system, thereby reducing β, and reduces confinement via parallel thermal conduction to electrodes[4]

Sheared flows, on the other hand, contribute to stabilization with fewer adverse effects eg.equilibrium force balance unaffected and $\langle\beta\rangle$ still of order unity. Theoretical analysis under the assumption of uniform flow shear, a dubious assumption which will be explored later since viscosity makes the assumption unrealizable, the kink mode can be stabilized by flow shear exceeding $\frac{dv_z}{dr} > 0.1kv_A$ where k is the axial wave number and v_A is the Alfven speed.[8]

Finally note when viscous effects, which can eliminate the flow shear, are to be considered, the viscous damping distance can be found $L_\mu = v_z n_i m_i L^2/\mu$ where the shear scale length is assumed to be the pinch radius $L = a$ Non-ideal effects are certainly to be considered; however, note that in experiments considered here, namely those based off of ZaP, it is found that the viscous damping distance is of order 2 meters at the plasma radius[4]

This provides a framework for a stable equilibrium, but let's now consider simulation and experimental findings to see how things hold up in less ideal environments.

5 A Sample of Recent Findings

The primary sources considered come from the ZaP device (including ZaP-HD) as well as the FuZE device. These experiments represent a progression in Z-pinch research by instituions in the US, noteably one of the primary improvements made in FuZE were coaxial electrodes in a similar geometry to the ZaP-HD upgrade which allowed a larger pinch current without degrading stability.

ZaP and ZaP-HD have demonstrated a small radius Z-pinch which is axially uniform with high magentic fields and temperature, suggesting a scalability to HED conditions. Notably the quiescent period in ZaP experiments corresponds to the period when the plasma is under a shear flow which exceeds the theoretical threshold. Displacements are large when the shear flow is off, small when shear flow is above threshold.[5]

Moreover, FuZE experiments have found stable equilibria on the order of thousands of Alfven times, much longer than the characteristic linear instability growth time.[2,5]

Nonlinear fluid simulations aligned with the FuZE configurations have shown that the theoretical scaling of neutron production with current, on the order I^{11}, still holds even when dynamcis such as initial plasma acceleration and pinch assembly are taken into account[2]

That being said, recent gyrokinetic and extended-MHD simulations of a SFS Z-pinch have shown mixed results. Geyko, Angus, and Dorf performed such simulations with the high order finite volume code COGENT, based off of density and temperature profiles from FuZE. Their code included effects such as the gyroviscous pressure tensor, diamagnetic electron and ion heat flux, and generalized ohm's law.

By analyzing linear growth rates they found the growth rate curve is similar to a Bennett profile, in that there is roll-over at high k_z. They note the importance of the direction of fluid shear and can range from being destabilizing to stabilizing, namely $\kappa > 0$ is ideal where κ is fluid shear. Furthermore they found in order for fluid flow shear to be comparable to E x B drift shear requires $\kappa \approx 1.5$ meaning superthermal (super-Alfvenic) shear. Experiments have not tested this regime so in order to investigate the observed stabilization the Geyko et al. investigated the nonlinear evolution of their system and found a nonlinear stability can be claimed due to short wavelength modes not propegating into the interior of their pinch.

To address long wavelength modes and drift effects the group then turned to extended-MHD models under the same code framework. Most of their findings were corroborated, with insights that FLR effects play an important role in linear mode stabilization, and that nonlinear effects are important since short-wavelength perturbations saturate on the periphery while long-wavelength perturbations penetrate and disrupt the pinch. Concluding again that under current parameters fluid shear $\kappa \approx 1.5$ is necessary to match the embedded guiding center drift shear.

One final finding, which merits further investigation wrt robustness, is that profile flattening can leverage the nonlinear stabilization mechanism by pushing instabilities to the periphery.

Their overall conclusion is that no sub-Alfvenic fluid flow shear is sufficient for stabilization of linear modes, nor relaxation of pressure gradients.[8]

6 Proposed Optimizations

Worth noting that some improvements to configurations such as Z-pinches are proposed all the time. Regarding Shear Flow Stabilization it is notable that theoretical results are typically based on uniform flow shear; however, experimental measurements do not find uniform flow shear. Also of note is that theoretical thresholds is sensitive to plasma equilibrium, based off of the Alfven speed, which is computed from the magnetic field measured from the outer electrode and extrapolated to the pinch radius.

Simulations have been done to investigate nonuniform flow shear. finding that shear must be located on the outer boundary of the plasma.[5]

Long story short regarding threshold values is that they've been found to agree with the underlying theory through experiment.[5]

7 Turbulence

Flow shear is found to suppress turbulence and turbulent transport, particularly when $\tau_s < \tau_N < \tau_D$ corresponding to shear straining time, turbulent correlation time, and domain time, respectively. It's found that if the driving source remains invariant across the shear flow, the turbulence will adjust itself to balance shear straining time and correlation time, $\tau_s = \tau_N$, decreasing τ_N which directly decreases turbulent energy. Enhancing transport barriers via shear flow has led to new records across a number of devices. Shear flow has also been shown to affect the complex phase angle between advection fluctuation and the advecting flow, producing a reduction in transport flux independent of amplitude reduction.[7] The invetigation of turbulence goes a bit beyond the scope of this report, but for further information please direct youself to the source for the previous paragraph, P. W. Terry.

8 Conclusion & Loose Threads

So we find some mixed results, mostly positive in that experiment and simulation seem to agree that shear flow stabilization successfully mitigates instabilities such that a quiescient Z-Pinch can be formed and thermonuclear neutrons have been produced.

Note the goose isn't cooked yet, there is still contention from siluation studies about the mechanism with which stabilization occurs, and a whole host of effects need to be investgated further as things heat up:

- Transient reversals in flow shear were observed in ZaP, conjectured to be caused by stagnation of the outer-radii flow on the electrode end wall[4]
- parameter scaling relations assume adiabaticity, ignoring shocks which may occur during initial formation of the pinch[4]
- Polytropic index should be investigated to predict parameters as current is increased[4]
- Instabilities exist which shear flow does not address, namely drift instabilties which will be more prevalent at higher current and drift speeds[4,2]
- plasma-neutral interactions at the pinch edge[2]
- finite gyroradius effects[2]
- plasma electrode interactions and impurity effects[2]
- decorrelation of turbulence, transport barriers, bifurcation points and self-criticality[7]

9 Sources

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