# From Reconnection to Relaxation: A Pedagogical Tale of Two Taylors

## or: The Physics Assumptions Behind the Color VG



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This talk focuses on:

- what is the connection between local reconnection and global relaxation?

- how do highly localized reconnection processes, for large Rm, Re, produce global self-organization and structure formation?

We attempt to:

- describe both magnetic fields and flows with similar concepts

- connect and relate to talks by H. Ji, D. Hughes, H. Li, O.D. Gurcan...

- describe self-organization principles

# Outline

i.) Preamble: → From Reconnection to Relaxation and Self-Organization

→ What 'Self-Organization' means

→ Why Principles are important

→ Examples of turbulent self-organization

→ Preview

ii.) Focus I: Relaxation in R.F.P. (J.B. Taylor)

→ RFP relaxation, pre-Taylor

→ Taylor Theory
- Summary
- Physics of helicity constraint + hypothesis
- Outcome and Shortcomings

→ Dynamics → Mean Field Theory
- Theoretical Perspective
- Pinch's Perspective
- Some open issues

→ Lessons Learned and Unanswered Questions

# Outline

iii.) Focus II: PV Transport and Homogenization (G.I. Taylor)

→ Shear Flow Formation by (Flux-Driven) Wave Turbulence

→ PV and its meaning; representative systems

→ Original Idea: G.I. Taylor, Phil. Trans, 1915, 'Eddy Motion in the Atmosphere'

- Eddy Viscosity, PV Transport and Flow Formation
- Application: Rayleigh from PV perspective

→ Relaxation: PV Homogenization (Prandtl, Batchelor, Rhines, Young)

- Basic Ideas
- Proof of PV Homogenization
- Time Scales
- Relation to Flux Expulsion
- Relation to Minimum Enstrophy states

# Outline

→ Does PV Homogenize in Zonal Flows?

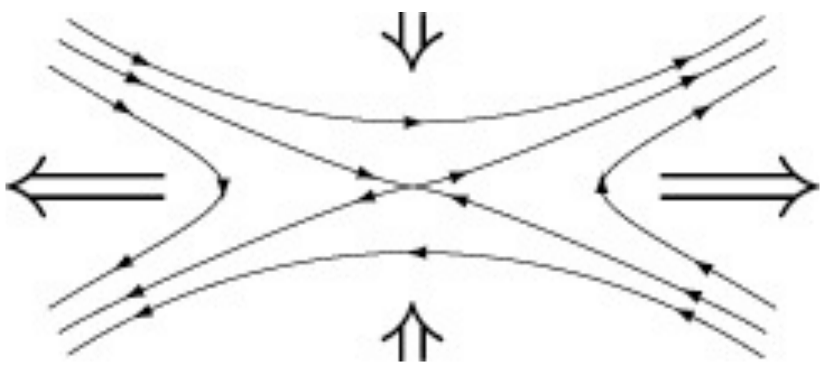
- Physical model and Ideas
- PV Transport and Potential Enstrophy Balance
- Momentum Theorems (Charney-Drazin) and Incomplete Homogenization
- RMP Effects
- $\mathbf{B}_0$ Effects
- Lessons Learned and Unanswered Questions

→ Discussion and General Lessons Learned

# I.) Preamble

→ From Reconnection to Relaxation

- Usually envision as localized event involving irreversibility, dissipation etc. at a singularity



S.-P.

$$V = V_A / Rm^{1/2}$$

- ??? - how describe global dynamics of relaxation and self-organization

→

- multiple, interacting/overlapping reconnection events

→ turbulence, stochastic lines, etc

# I.) Preamble, cont'd

→ What does 'Self-Organization' mean?

- context: driven, dissipative, open system
- turbulence/stochasticity - multiple reconnection states
- Profile state (resilient, stiff) attractors
- usually, multiple energy channels possible
- bifurcations between attractor states possible
- attractor states macroscopically stable, though may support microturbulence

→ Elements of Theory

- universality (or claims thereof)
- coarse graining - i.e., diffusion
- constraint release - i.e., relaxation of freezing-in law
- selective decay hypothesis

| RFP | Tokamak |
|---|---|
| Taylor/BFM | Stiff core + edge |
| $I_p$ → $P_{OH}$<br>$I_p$ → B profile | $Q$ → Turbulence<br>Turbulence ↓ ↑ Flows |
| axisymmetric → helical OH | L → H |
| nearly marginal $m=1$ 's<br>+ resistive interchange +... | ITG, CTEM, ...<br>Issue: ELMs?! (domain limited) |

- Universality:

Taylor State (Clear)

$$H_M = \int d^3x \mathbf{A} \cdot \mathbf{B}$$

only constraint

Magnetic energy dissipated as $H_M$ conserved

Profile Consistency (especially pedestal) (soft)

PV mixed, subject dynamical constraints

Enstrophy (Turbulence) mixed, dissipated, as macroscopic flow emerges

# Why Principles?

→ INSIGHT

→ Physical ideas necessary to guide both physical and digital experiments

→ Principles + Reduced Models required to extract and synthesize lessons from case-by-case analysis

→ Principles guide approach to problem reduction

# Examples of Self-Organization Principles

→ Turbulent Pipe Flow: (Prandtl → She)

$$\sigma = -\nu_T \frac{\partial \langle v_y \rangle}{\partial x} \qquad \nu_T \sim v_* x \qquad \Rightarrow \langle v_y \rangle \sim v_* \ln x$$

Streamwise Momentum undergoes scale invariant mixing

→ Magnetic Relaxation: (Woltjer-Taylor)

(RFP, etc)

(Focus 1) Minimize $E_M$ at conserved global $H_M$ $\Rightarrow$ Force-Free RFP profiles

→ PV Homogenization/Minimum Enstrophy: (Taylor, Prandtl, Batchelor, Bretherton, ...)

(Focus 2)

→ PV tends to mix and homogenize

→ Flow structures emergent from selective decay of potential enstrophy relative energy

→ Shakura-Sunyaev Accretion

→ disk accretion enabled by outward viscous angular momentum flux

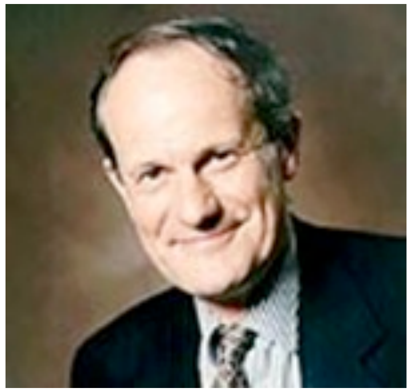
# Preview

- Will show many commonalities - though NOT isomorphism - of magnetic and flow self-organization

- Will attempt to expose numerous assumptions in theories thereof

| | Magnetic (JB) | Flow (GI) |
|---|---|---|
| concept | topology | symmetry |
| process | turbulent reconnection | PV mixing |
| players | tearing modes, Alfven waves | drift wave turbulence |
| mean field | EMF = $\langle \tilde{v} \times \tilde{B} \rangle$ | PV Flux = $\langle \tilde{v}_r \tilde{q} \rangle$ |
| constraint | $\int d^3x \mathbf{A} \cdot \mathbf{B}$ conservation | Potential Enstrophy balance |
| NL | Helicity Density Flux | Pseudomomentum Flux |
| outcome | B-profiles | zonal flow |

## II.) Focus I - Magnetic Relaxation



(Derek C Robinson)

→ Prototype of RFP's: Zeta (UK: late 50's - early 60's )

- toroidal pinch = vessel + gas + transformer
- initial results → violent macro-instability, short life time
- weak $B_T$ → stabilized pinch ↔ sausage instability eliminated
- $I_p > Ip, crit$ ( $\theta > 1+$ ) → access to "Quiescent Period"

→ Properties of Quiescent Period:

- macrostability - reduced fluctuations
- $\tau_E \sim 1\ msec$ $T_e \sim 150eV$
- $B_T(a) < 0$ → reversal

→ Quiescent Period is origin of RFP

# Further Developments

- Fluctuation studies:

turbulence = $m = 1$ kink-tearing → tend toward force-free state

resistive interchange, ...

- Force-Free Bessel Function Model

$$B_\theta = B_0 J_1(\mu r) \qquad B_z = B_0 J_0(\mu r)$$

$$\mathbf{J} = \alpha \mathbf{B}$$

observed to correlate well with observed B structure

- L. Woltjer (1958) : Force-Free Fields at constant $\alpha$

→ follows from minimized $E_M$ at conserved $\int d^3x \mathbf{A} \cdot \mathbf{B}$

- steady, albeit modest, improvement in RFP performance, operational space

→ Needed: Unifying Principle

# Theory of Turbulent Relaxation (J.B. Taylor, 1974)

→ hypothesize that relaxed state minimizes magnetic energy subject to constant global magnetic helicity

i.e. profiles follow from: $$\delta \left[ \int d^3x \frac{B^2}{8\pi} + \lambda \int d^3x \mathbf{A} \cdot \mathbf{B} \right] = 0$$

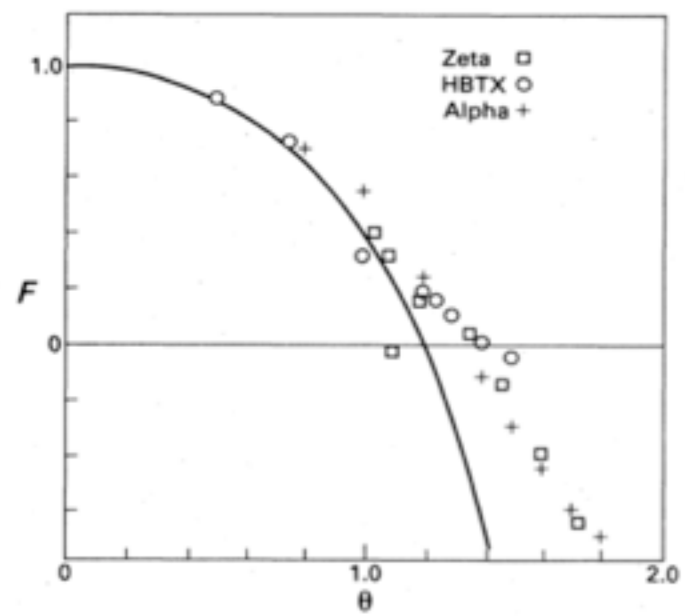
$$\Rightarrow \ \nabla \times \mathbf{B} = \mu \mathbf{B} \ \ ; \ \ J_{\|}/B = \frac{\mathbf{J} \cdot \mathbf{B}}{B^2} = const$$

Taylor state is:

- force free
- flat/homogenized $J_{\|}/B$
- recovers BFM, with reversal for $\theta = \dfrac{2I_p}{aB_0} > 1.2$

- Works amazingly well

# Result:




$$\theta = \mu a/2 = \frac{2I_p}{aB_0}$$

$$F = B_{z,wall}/\langle B \rangle$$

and numerous other success stories

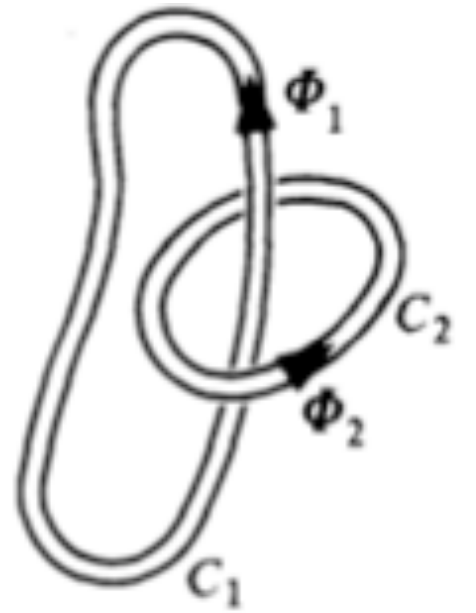
→ Questions:

- what is magnetic helicity and what does it mean?
- why only global magnetic helicity as constraint?
- Theory predicts end state → what can be said about dynamics?
- What does the pinch say about dynamics?

→ Central Issue: Origin of Irreversibility

# Magnetic helicity - what is it?

- consider two linked, closed flux tubes




Tube 1: Flux $\phi_1$ , contour $C_1$

Tube 2: Flux $\phi_2$ , contour $C_2$

if consider tube 1:

$$H_M^1 = \int_{V_1} d^3x \mathbf{A}\cdot\mathbf{B} = \oint_{C_1} dl \int_{A_1} dS \mathbf{A}\cdot\mathbf{B}$$
$$= \oint_{C_1} d\mathbf{l}_1 \cdot \mathbf{A} \int_{A_1} d\mathbf{a}\cdot\mathbf{B}$$
$$= \phi_1 \oint_{C_1} d\mathbf{l}_1 \cdot \mathbf{A} = \phi_1\phi_2$$

similarly for tube 2: $H_M^2 = \phi_1\phi_2$

so $H_M = 2\phi_1\phi_2$

generally : $H_M = \pm 2n\phi_1\phi_2$

- Magnetic helicity measures self-linkage of magnetic configuration

- conserved in ideal MHD - topological invariant

$$\frac{d}{dt} H_M = -2\eta c \int d^3x \mathbf{J} \cdot \mathbf{B}$$

- consequence of Ohm's Law structure, only

N.B.

- can attribute a finite helicity to each closed flux tube with non-constant $q(r)$

- in ideal MHD → $\infty$ number of tubes in pinch. Can assign infinitesimal tube to each field line

- $\infty$ number of conserved helicity invariants

  → Follows from freezing in

## Question:

How many magnetic field lines in the universe?

(E. Fermi to M.N. Rosenbluth, oral exam at U. Chicago, late 1940's...)

# Why Global helicity, Only?

- in ideal plasma, helicity conserved for each line, tube

i.e. $\mathbf{J} = \mu(\alpha, \beta)\mathbf{B}$ $\mu(\alpha', \beta') \neq \mu(\alpha, \beta)$

- Turbulent mixing eradicates identity of individual flux tubes, lines!

i.e.

- if turbulence s/t field lines stochastic, then '1field line' fills pinch.

  1 line ↔ 1tube → only global helicity meaningful.

- in turbulent resistive plasma, reconnection occurs on all scales, but: $\tau_R \sim l^{\alpha}$ $\alpha > 0$

  ( $\alpha = 3/2$ for S-P reconnection)

  Thus larger tubes persist longer. Global flux tube most robust

- selective decay: absolute equilibrium stat. mech. suggests possibility of inverse cascade of magnetic helicity (Frisch '75) → large scale helicity most rugged.

# Comments and Caveats

→ Taylor's conjecture that global helicity is most rugged invariant remains a conjecture

→ unproven in any rigorous sense

→ many attempts to expand/supplement the Taylor conjecture have had little lasting impact (apologies to some present....)

→ Most plausible argument for global $H_M$ is stochastization of field lines → forces confinement penalty. No free lunch!

→ Bottom Line:

- Taylor theory, simple and successful

- but, no dynamical insight!

# Dynamics I:

- The question of Dynamics brings us to mean field theory (c.f. Moffat '78 and an infinity of others - see D. Hughes, Thursday Lecture)

- Mean Field Theory $\rightarrow$ how represent $\langle \tilde{v} \times \tilde{B} \rangle$ ?

  $\rightarrow$ how relate to relaxation?

- Caveat: - MFT assumes fluctuations are small and quasi-Gaussian. They are often NOT

  - MFT is often very useful, but often fails miserably

- Structural Approach (Boozer): (plasma frame)

$$\langle \mathbf{E} \rangle = \eta \langle \mathbf{J} \rangle + \langle \mathbf{S} \rangle$$

$\rightarrow$ something $\rightarrow$ related to $\langle \tilde{v} \times \tilde{B} \rangle$

$\langle \mathbf{S} \rangle$ conserves $H_M$

$\langle \mathbf{S} \rangle$ dissipates $E_M$

Note this is ad-hoc, forcing $\langle \mathbf{S} \rangle$ to fit the conjecture. Not systematic, in sense of perturbation theory

Now

$$\partial_t H_M = -2c\eta \int d^3x \langle \mathbf{J} \cdot \mathbf{B} \rangle - 2c \int d^3x \langle \mathbf{S} \cdot \mathbf{B} \rangle$$

$$\therefore \langle \mathbf{S} \rangle = \frac{\mathbf{B}}{B^2} \nabla \cdot \mathbf{\Gamma}_H$$

Conservation $H_M$ $\rightarrow$ $\langle S \rangle \sim \nabla \cdot$ (Helicity flux)

$$\partial_t \int d^3x \frac{B^2}{8\pi} = -\int d^3x \left[ \eta J^2 - \mathbf{\Gamma}_H \cdot \nabla \frac{\langle \mathbf{J} \rangle \cdot \mathbf{B}}{B^2} \right]$$

so

$$\mathbf{\Gamma}_H = -\lambda \nabla (J_{||}/B)$$ , to dissipate $E_M$

→ simplest form consistent with Taylor hypothesis

→ turbulent hyper-resistivity $\lambda = \lambda[\langle \tilde{B}^2 \rangle]$ - can derive from QLT

→ Relaxed state: $\nabla (J_{||}/B) \rightarrow 0$ homogenized current → flux vanishes

# Dynamics II: The Pinch's Perspective

- Boozer model not based on fluctuation structure, dynamics

- Aspects of hyper-resistivity do enter, but so do other effects

→ Point: Dominant fluctuations controlling relaxation are m=1 tearing modes resonant in core → global structure

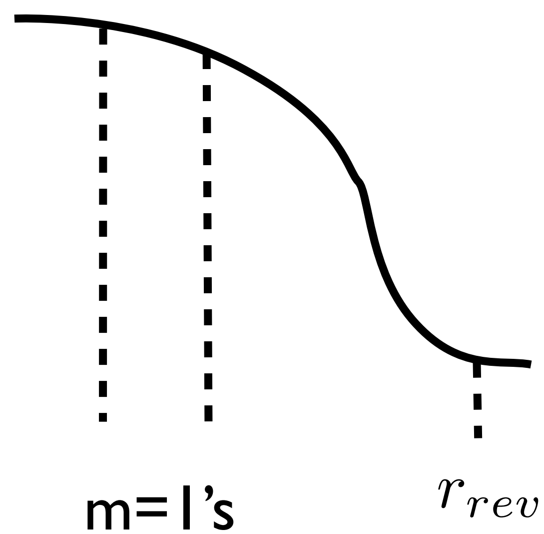
→ Issue: What drives reversal $B_z$ near boundary?

Approach: QL $\langle \tilde{v} \times \tilde{B} \rangle$ in MHD exterior - exercise: derive!

$$\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle \cong \sum_k |\gamma_k| \frac{R}{r} (q_{res} - q(r)) \langle B_\theta \rangle \partial_r (|\tilde{\xi}_r|_k^2)$$

i.e. $\langle J_\theta \rangle$ driven opposite $\langle B_\theta \rangle$ → drives/sustains reversal

→ What of irreversibility - i.e. how is kink-driven reversal 'locked-in'?




→ drive $J_{\|}/B$ flattening, so higher n's destabilized by relaxation front

→ global scattering → propagating reconnection front

m=1, n    m=1, n+1 → m=0, n=1 → driven current sheet, at $r_{rev}$

(difference beat)

sum beat { m=2, 2n+1

but then m=1, n+2 driven → tearing activity, and relaxation region, broadens

→ Bottom Line: How Pinch 'Taylors itself' remains unclear, in detail

# Summary of Magnetic Relaxation

concept: topology

process: stochastization of fields, turbulent reconnection

constraint released: local helicity

players: tearing modes

Mean Field: EMF = $\langle \tilde{v} \times \tilde{B} \rangle$

Global Constraint: $\int d^3x \mathbf{A} \cdot \mathbf{B}$

NL: Helicity Density Flux

Outcome: B-Profile

Shortcoming: Rates, confinement $\rightarrow$ turbulent transport

## Focus II: Potential Vorticity Mixing $\leftrightarrow$ Iso-vorticity Contour Reconnection

→ Prandtl-Batchelor Theorem and PV Homogenization

→ Self-Organization of Zonal Flows

# PV and Its Meaning: Representative Systems

## The Fundamentals

- Kelvin's Theorem for rotating system

$$\underbrace{\omega}_{\text{relative}} \to \omega + \underbrace{2\boldsymbol{\Omega}}_{\text{planetary}} \quad \longrightarrow \quad \oint \mathbf{v}\cdot d\mathbf{l} = \int d\mathbf{a}\cdot(\omega + 2\Omega) \equiv C$$

$\dot{C} = 0$ , to viscosity (vortex reconnection)

- $Ro = V/(2\Omega L) \ll 1 \quad \rightarrow \quad \mathbf{V} \cong -\nabla_\perp p \times \hat{z}/(2\Omega)$ geostrophic balance

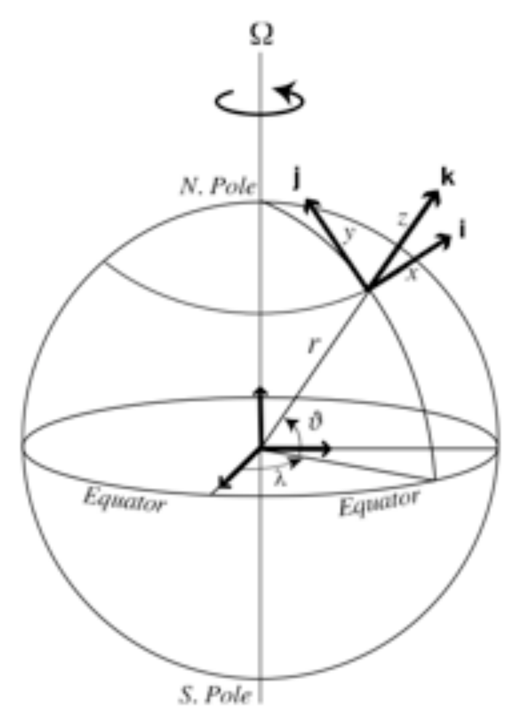
→ 2D dynamics

- Displacement on beta plane

$$\dot{C} = 0 \quad \rightarrow \quad \frac{d}{dt}\omega \cong -\frac{2\Omega}{A}\sin\theta_0 \frac{dA}{dt}$$

$$= -2\Omega\frac{d\theta}{dt} = -\beta V_y$$

$\omega = \nabla^2\phi,\quad \beta = 2\Omega\sin\theta_0/R$



# Fundamentals II

- Q.G. equation $\frac{d}{dt}(\omega + \beta y) = 0$

n.b. topography

- Locally Conserved PV $q = \omega + \beta y$

$$q = \omega/H + \beta y$$

- Latitudinal displacement → change in relative vorticity

- Linear consequence → Rossby Wave

$$\omega = -\beta k_x/k^2$$

observe: $v_{g,y} = 2\beta \underline{k_x k_y}/(k^2)^2$

↳ Rossby wave intimately connected to momentum transport

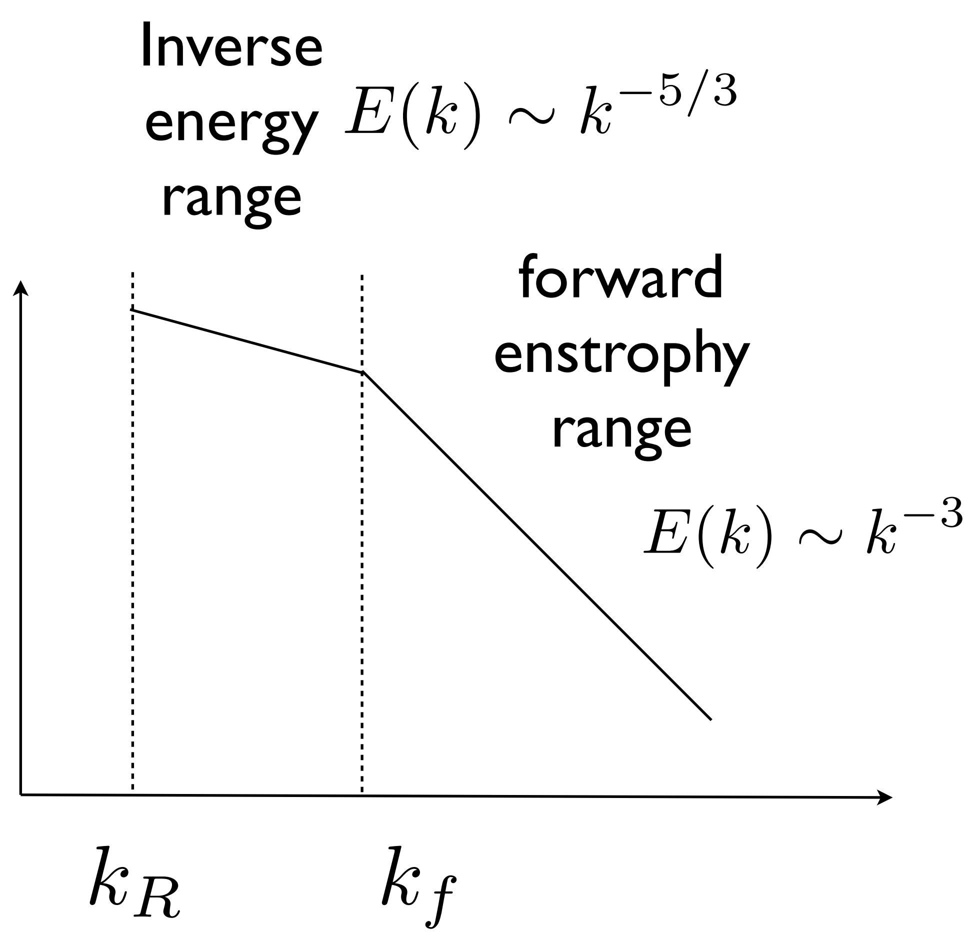
- Latitudinal PV Flux → circulation

# - Obligatory re: 2D Fluid

- $\omega$ Fundamental: $$\partial_t \omega = \nabla \times (\mathbf{V} \times \omega)$$

$$\frac{d}{dt}\frac{\omega}{\rho} = \frac{\omega}{\rho} \cdot \nabla \mathbf{V}$$ → Stretching

- 2D $d\omega/dt = 0$ → $$E = \langle v^2 \rangle$$ $$\Omega = \langle \omega^2 \rangle$$ conserved




How?

$$\partial_t \langle \Delta k^2 \rangle_E > 0$$ with $$\dot{E} = \dot{\Omega} = 0$$

$$\partial_t \langle \Delta k^2 \rangle_E = -\partial_t \bar{k}_E^2$$

dual cascade
$$\begin{cases} \partial_t \bar{k}_E^{\,2} < 0 \\ \partial_t \bar{k}_\Omega^{\,2} > 0 \end{cases}$$

→ large scale accumulation

→ flow to small scale dissipation

→ Isn't this Meeting about Plasma?

→ 2 Simple Models

a.) Hasegawa-Wakatani (collisional drift inst.)

b.) Hasegawa-Mima (DW)

a.) $\mathbf{V} = \frac{c}{B}\hat{z} \times \nabla\phi + \mathbf{V}_{pol}$

$\sim (\omega/\Omega)$

$L > \lambda_D$ → $\nabla \cdot \mathbf{J} = 0$ → $\nabla_\perp \cdot \mathbf{J}_\perp = -\nabla_\| J_\|$

$J_\perp = n|e|V_{pol}^{(i)}$

$J_\| : \eta J_\| = -(1/c)\partial_t A_\| - \nabla_\| \phi + \nabla_\| p_e$

e.s.

b.) $dn_e/dt = 0$

→ $\frac{dn_e}{dt} + \frac{\nabla_\| J_\|}{-n_0|e|} = 0$

n.b.

MHD: $\partial_t A_\|$ v.s. $\nabla_\| \phi$

DW: $\nabla_\| p_e$ v.s. $\nabla_\| \phi$

## So H-W

$$\rho_s^2 \frac{d}{dt} \nabla^2 \hat{\phi} = -D_{\|} \nabla_{\|}^2 (\hat{\phi} - \hat{n}/n_0) + \nu \nabla^2 \nabla^2 \hat{\phi}$$

$$\frac{d}{dt} n - D_0 \nabla^2 \hat{n} = -D_{\|} \nabla_{\|}^2 (\hat{\phi} - \hat{n}/n_0)$$

$D_{\|} k_{\|}^2 / \omega$ is key parameter

n.b. $\mathrm{PV} = n - \rho_s^2 \nabla^2 \phi$ $\quad \frac{d}{dt}(\mathrm{PV}) = 0$

→ total density

b.) $D_{\|} k_{\|}^2 / \omega \gg 1 \rightarrow \hat{n}/n_0 \sim e\hat{\phi}/T_e \qquad (m, n \neq 0)$

$$\frac{d}{dt}(\phi - \rho_s^2 \nabla^2 \phi) + v_* \partial_y \phi = 0$$ → H-M

n.b. $\mathrm{PV} = \phi - \rho_s^2 \nabla^2 \phi + \ln n_0(x)$

An infinity of models follow:

- MHD: ideal ballooning
  resistive → RBM

- HW + $A_{\|}$: drift - Alfven

- HW + curv. : drift - RBM

- HM + curv. + Ti: Fluid ITG

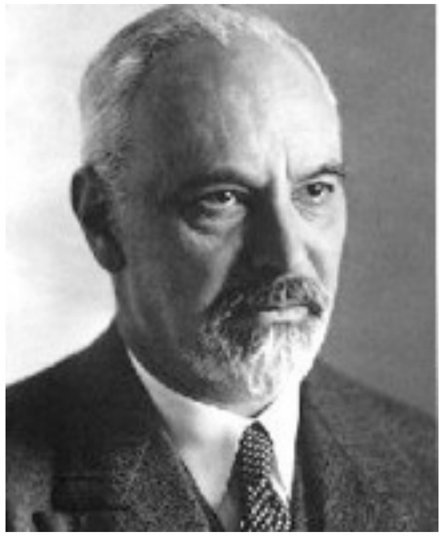
- gyro-fluids

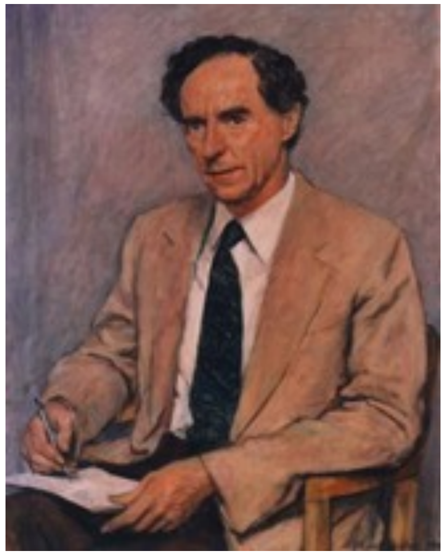
- GK

N.B.: Most Key advances appeared in consideration of simplest possible models

# Homogenization Theory (Prandtl, Batchelor, Rhines, Young)

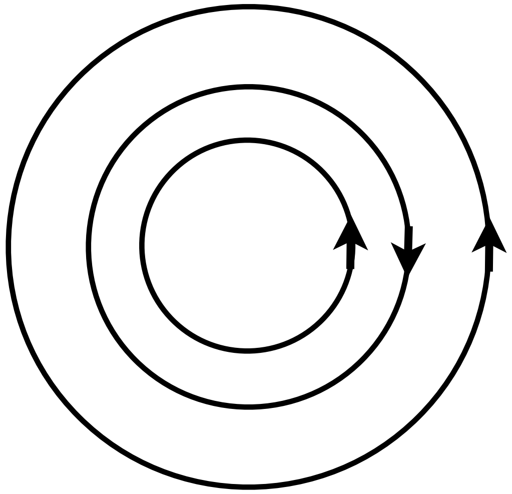
$$\partial_t q + \nabla\phi \times \hat{z} \cdot \nabla q = \nu \nabla^2 q$$





Now: $t \to \infty$ $\partial_t q \to 0$

For $\nu = 0$ $q = q(\phi)$



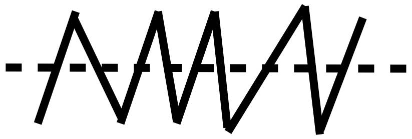
→ $q = q(\phi)$ is arbitrary solution

→ can develop arbitrary fine scale $q = q(\phi)$

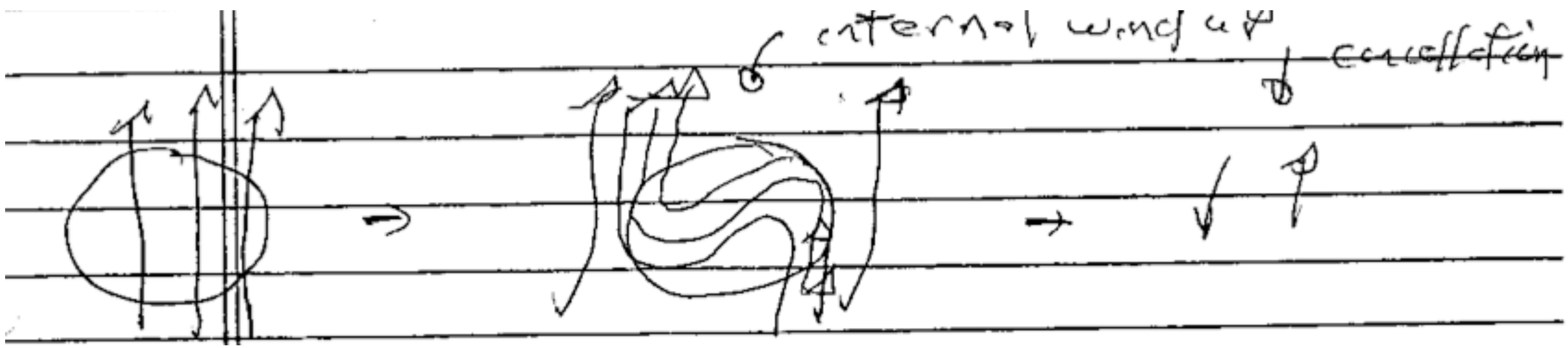
→ closed stream lines, $\nu = 0$

→ no irreversibility

i.e.



Now $\nu \neq 0$




→ non-diffusive stretching produces arbitrary fine scale structure

→ for small, but finite $\nu$ , instead of fine scale structure, must have:

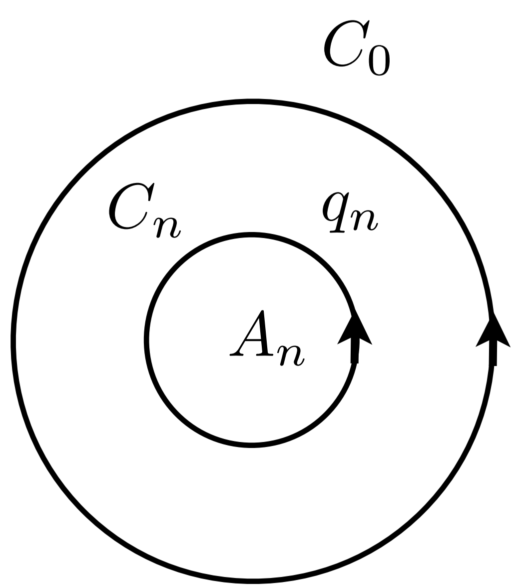
$q(\phi) \rightarrow const$ $\quad t \rightarrow \infty$ $\quad$ small $\nu$ → global behavior

i.e. finite $\nu$ at large $Re$ → PV homogenization

analogy in MHD? →Flux Expulsion

## Prandtl - Batchelor Theorem:

Consider a region of 2D incompressible flow (i.e. vorticity advection) enclosed by closed streamline $C_0$ .
Then, if diffusive dissipation, i.e. $\partial_t q + \nabla\phi \times \hat{z} \cdot \nabla q = \nu \nabla^2 q$
then vorticity → uniform (homogenization), as $t \to \infty$
within $C_0$




→ underpins notion of PV mixing → basic trend

→ fundamental to selective decay to minimum enstrophy state in 2D fluids (analogue of Taylor hypothesis)

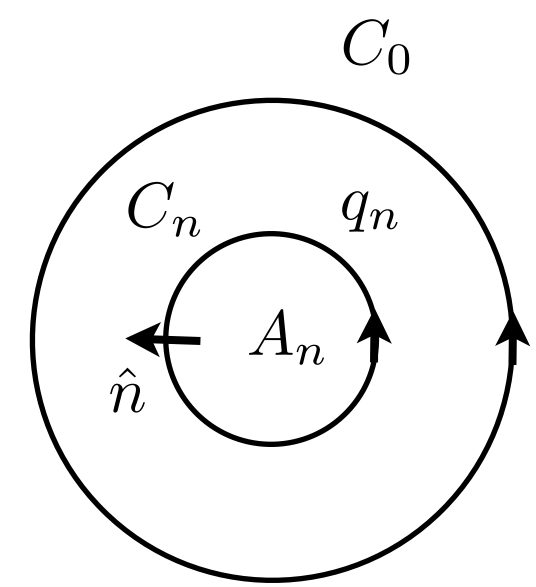
Proof:

$$\int_{A_n} \nabla \cdot (\mathbf{v}q) = 0$$ (closed streamlines)

$$0 = \int_{A_n} \nabla \cdot (\nu \nabla q)$$

$$= \nu \int_{C_n} dl \hat{n} \cdot \nabla q$$

(form of dissipation relevant!)




$C_0 \equiv$ bounding streamline

For $q = q(\phi)$

$$0 = \nu \int_{C_n} dl \hat{n} \cdot \nabla \phi_n \frac{\delta q}{\delta \phi_n}$$

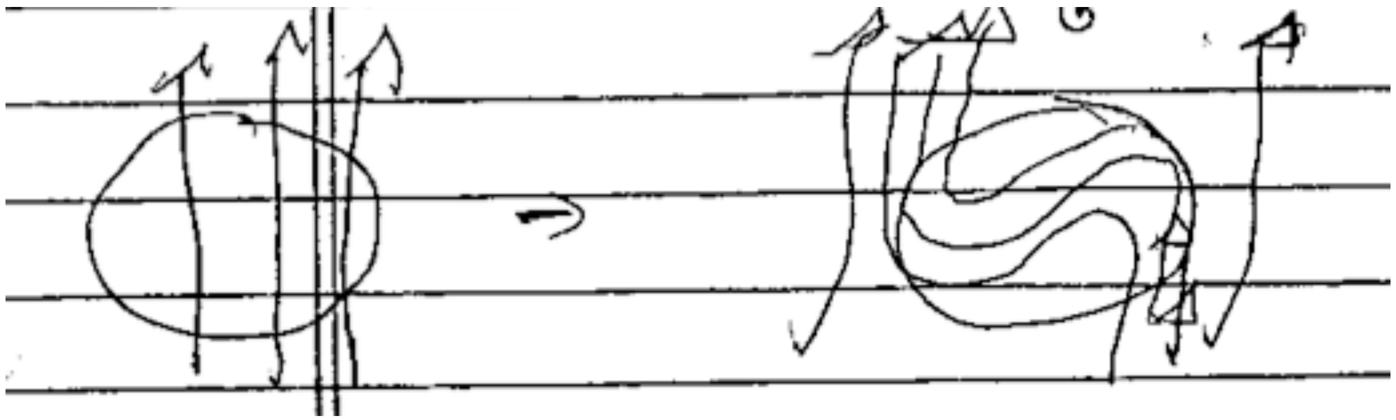
$$= \nu \frac{\delta q}{\delta \phi_n} \int_{C_n} dl \hat{n} \cdot \nabla \phi_n$$

$$\therefore 0 = \nu \frac{\delta q}{\delta \phi_n} \Gamma_n$$

$\therefore \dfrac{\delta q}{\delta \phi_n} = 0$ → q homogenized, within $C_0$

→ q' tends to flatten!

How long to homogenize? $\leftrightarrow$ What are the time scales?



Key: Differential Rotation within Eddy

Key: synergism between shear and diffusion

$$1/\tau_{mix} \sim 1/\tau_c (Re)^{-1/3}$$

$\tau_c \equiv$ circulation time

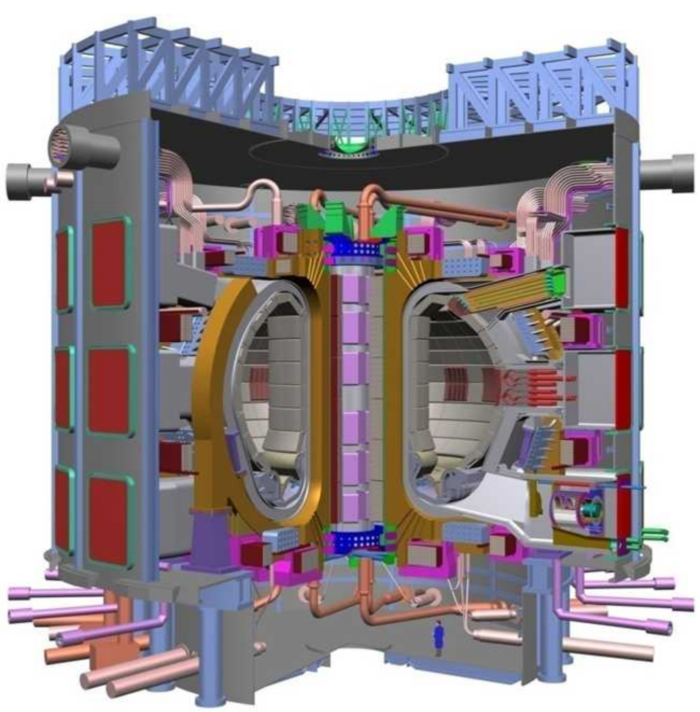
PV homogenization occurs on hybrid decorrelation rate

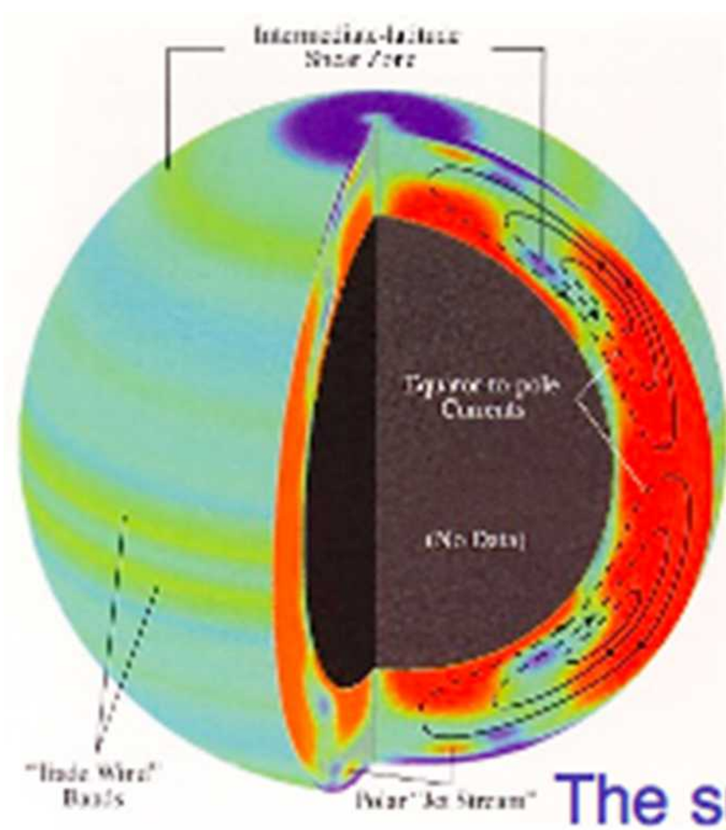
but $\tau_{mix} \ll \tau_D$ for $Re \gg 1$ $\longrightarrow$ time to homogenize is finite

Point of the theorem is global impact of small dissipation - akin Taylor

# Preamble I

- Zonal Flows Ubiquitous for:
  - ~ 2D fluids / plasmas $R_0 < 1$
    Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification
  - Ex: MFE devices, giant planets, stars…





The sun

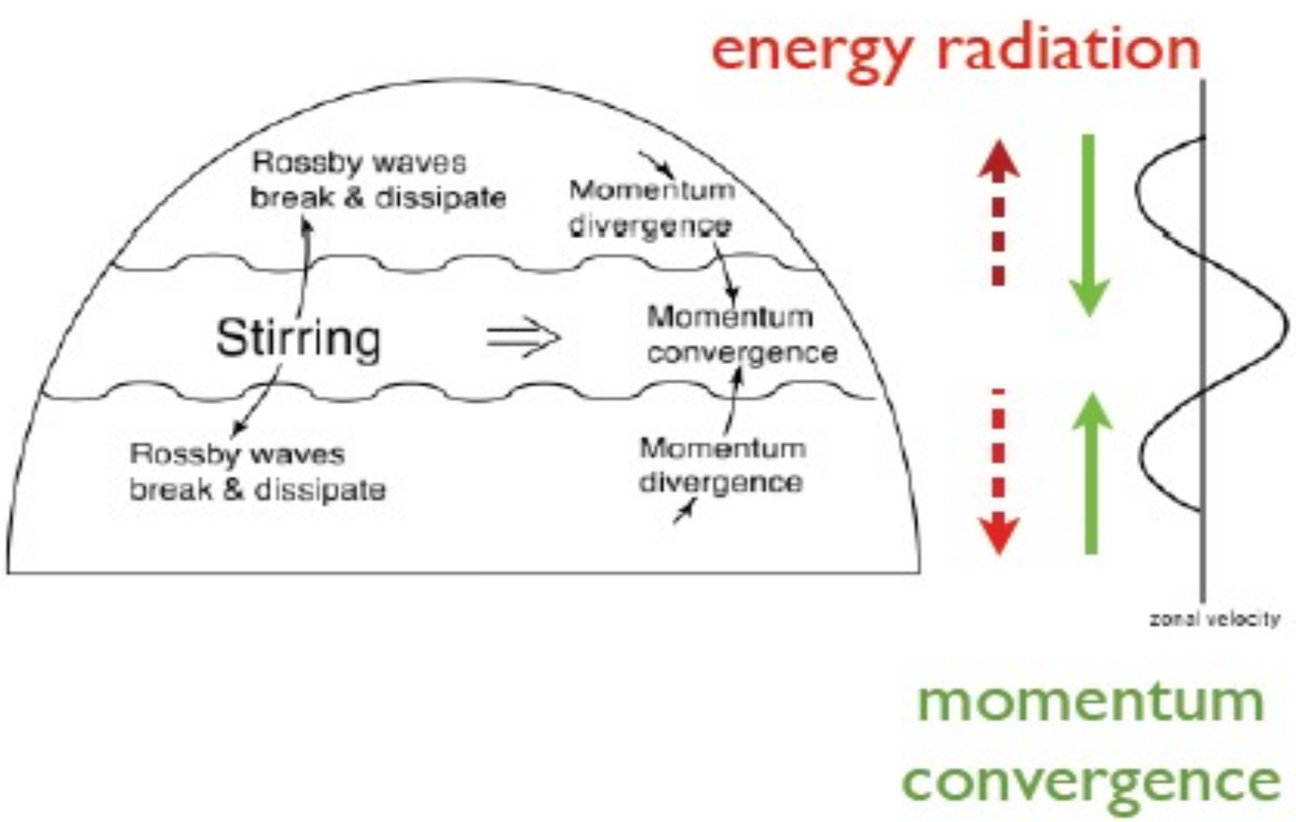
# Preamble II

- What is a Zonal Flow?
  - $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

# Heuristics of Zonal Flows a):

## Simplest Possible Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:




Rossby Wave:

$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

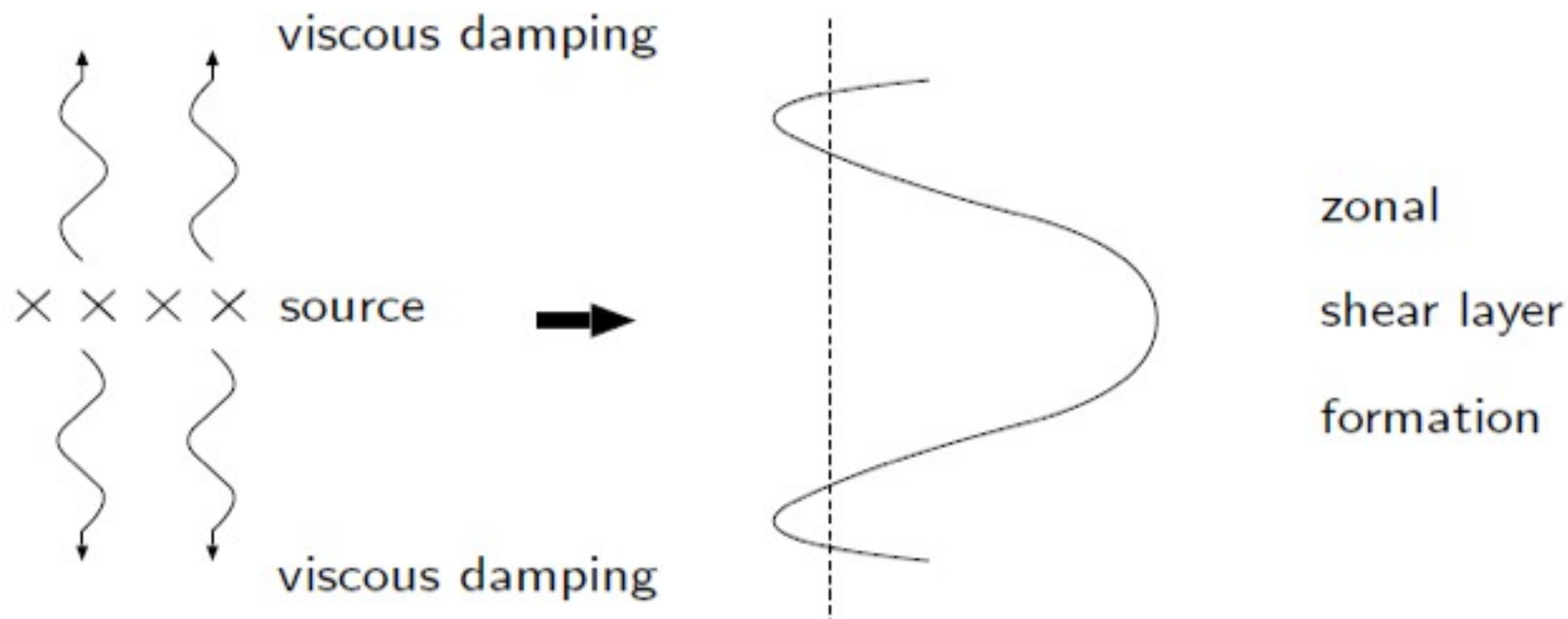
$$v_{gy} = 2\beta \frac{k_x k_y}{k_\perp^2} \qquad \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_k -k_x k_y \left|\hat{\phi}_{\vec{k}}\right|^2$$

$$\therefore \quad v_{gy} v_{phy} < 0$$

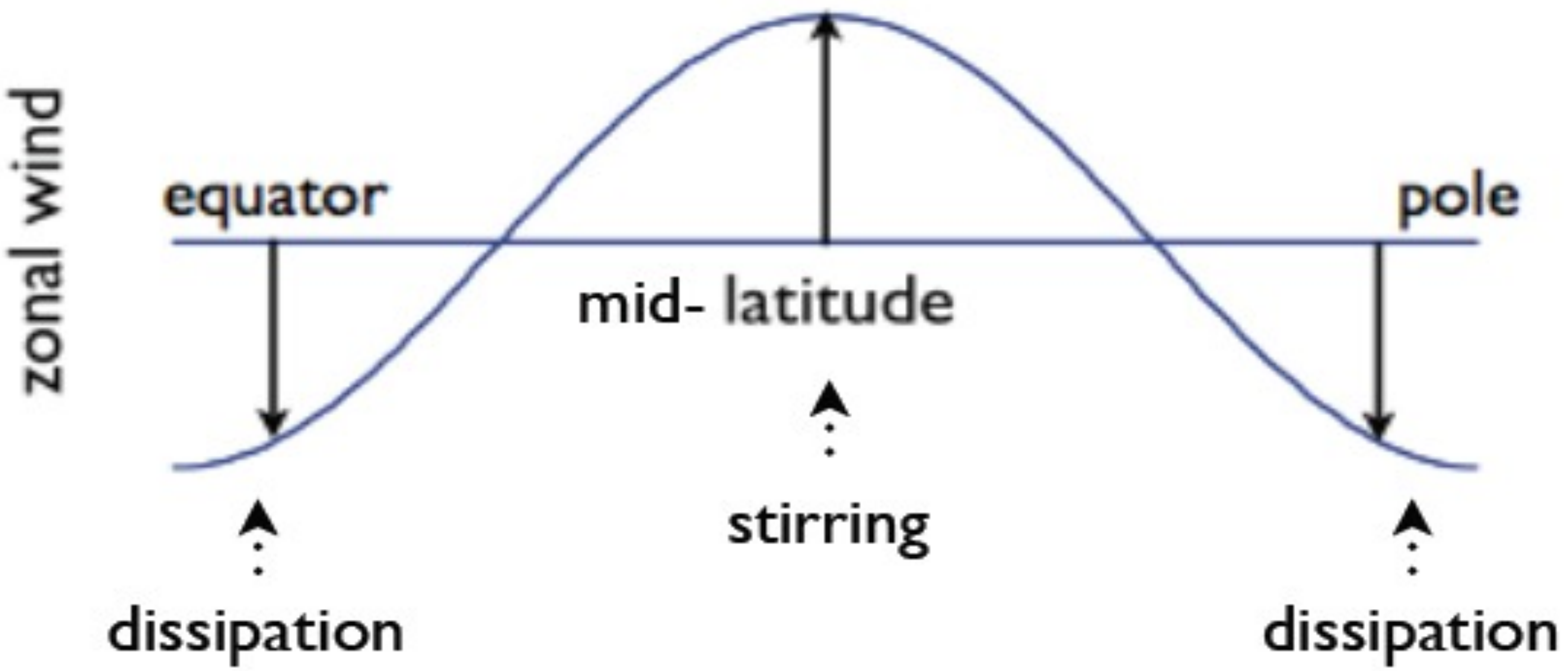
$\rightarrow$ Backward wave!

$\Rightarrow$ Momentum convergence at stirring location

- …"the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves $\Rightarrow$ incoming wave momentum flux




- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena
    $\rightarrow$ both `negative diffusion' phenomena




Key Point: Finite Flow Structure requires *separation* of

excitation and dissipation regions.

=> Spatial structure and wave propagation within are central.

→ momentum transport by waves

Key Elements:

- Waves $\rightarrow$ propagation transports momentum $\leftrightarrow$ stresses

  $\rightarrow$ modest-weak turbulence

- vorticity transport $\rightarrow$ momentum transport $\rightarrow$ Reynolds force

  $\rightarrow$ the Taylor Identity

* - Irreversibility $\rightarrow$ outgoing wave boundary conditions

- symmetry breaking $\rightarrow$ direction, boundary condition

  $\rightarrow \beta$

- Separation of forcing, damping regions

  $\rightarrow$ need damping region broads than source region

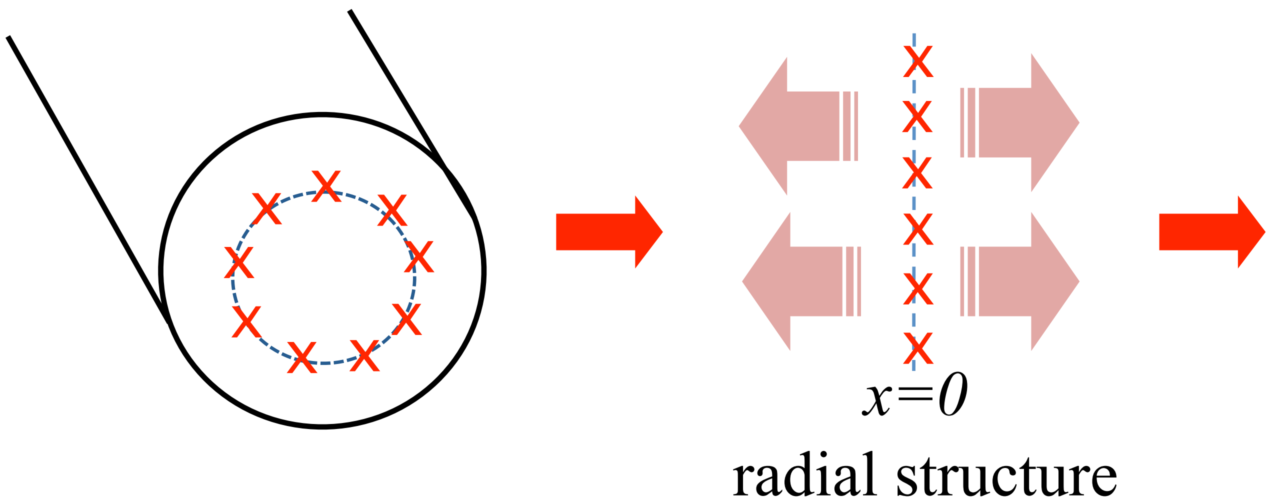
  $\rightarrow$ akin intensity profile...

All have obvious MFE counterparts...

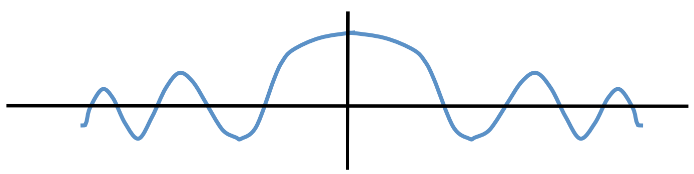
# Heuristics of Zonal Flows b.)

## 2) MFE perspective on Wave Transport in DW Turbulence

- localized source/instability drive intrinsic to drift wave structure




– couple to damping ↔ outgoing wave
i.e. Pearlstein-Berk eigenfunction

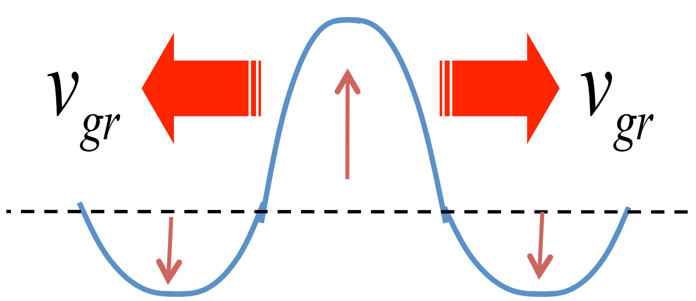


$x > 0 \;\Rightarrow\; v_{gr} > 0$

– $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$ $\qquad v_* < 0 \to k_r k_\theta > 0$

– $\langle v_{rE} v_{\theta E} \rangle = -\dfrac{c^2}{B^2} |\phi_{\vec{k}}|^2 \, k_r k_\theta < 0$

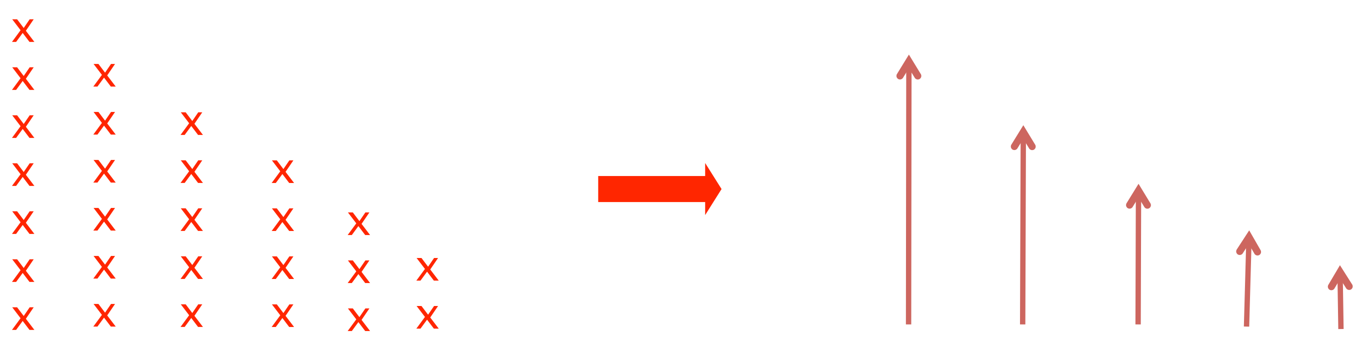
- outgoing wave energy flux → incoming wave momentum flux
  → counter flow spin-up!




- zonal flow layers form at excitation regions

# Heuristics of Zonal Flows b.) cont'd

- So, if spectral intensity gradient → net shear flow → mean shear formation



$$S_r = v_{gr}\varepsilon \cong -\frac{2k_r k_\theta V_t \rho_*^2}{\left(1 + k_\perp^2 \rho^2\right)}\varepsilon$$

$$\langle \tilde{v}_r \tilde{v}_\theta \rangle \approx \sum_k -k_r k_\theta \left|\phi_{\vec{k}}\right|^2$$

- Reynolds stress proportional radial wave energy flux $\vec{S}$, mode propagation physics (Diamond, Kim '91)
- Equivalently: $\partial_t E + \nabla \cdot \mathbf{S} + (\omega \mathrm{Im}\omega)E = 0$ (Wave Energy Theorem)
  - ∴ Wave dissipation coupling sets Reynolds force at stationarity
- Interplay of drift wave and ZF drive originates in mode dielectric
- Generic mechanism…

# Towards Calculating Something: Revisiting Rayleigh from PV Perspective

– G.I. Taylor's take on Rayleigh criterion

- consider effect on (zonal) flow by displacement of PV: $\delta y$

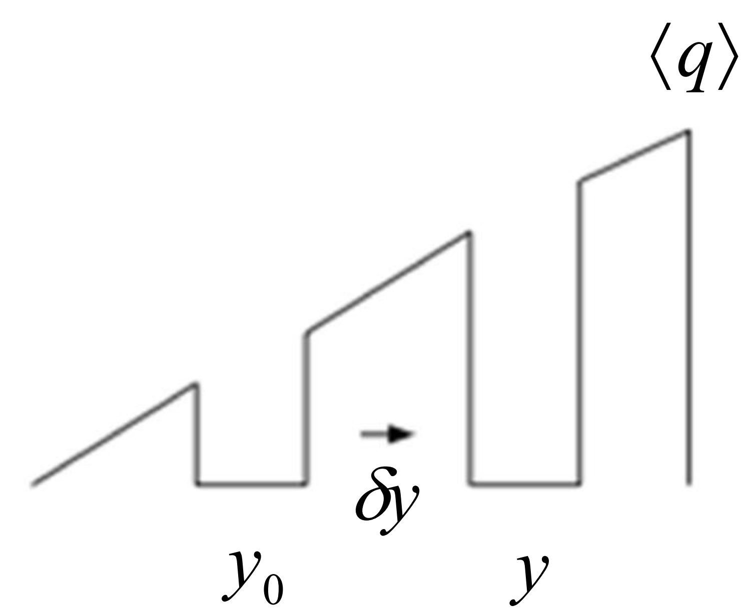
$$\frac{\partial}{\partial t}\langle v_x\rangle = \langle \tilde{v}_y \tilde{q}\rangle$$

$$\tilde{q} = (\text{PV of vorticity blob at y}) - (\text{mean PV at y})$$

$$\langle q(y)\rangle = \langle q(y_0)\rangle + (y - y_0)\left.\frac{d\langle q\rangle}{dy}\right|_{y_0}$$

Small displacement

$$\therefore \frac{\partial}{\partial t}\langle v_x\rangle = -\langle \tilde{v}_y \delta y\rangle \frac{d\langle q\rangle}{dy} = -\left(\partial_t \frac{\langle \tilde{\varepsilon}^2\rangle}{2}\frac{d\langle q\rangle}{dy}\right)$$




Flow driven by PV Flux

So, for instability
$$\begin{cases} \partial_t \langle \tilde{\varepsilon}^2 \rangle > 0 & \text{; growing displacement} \\ \dfrac{\partial}{\partial t} \displaystyle\int_{-a}^{a} dy \langle v_x \rangle = 0 & \text{; momentum conservation} \end{cases}$$

$$-\int_{-a}^{a} dy \left( \partial_t \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \right) \frac{d\langle q \rangle}{dy} = 0$$

$\dfrac{d\langle q \rangle}{dy}$ must change sign within flow interval

$\Rightarrow$ inflection point

also,

$$\frac{\partial}{\partial t} \left\{ \langle v_x \rangle + \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \frac{d\langle q \rangle}{dy} \right\} = 0 \qquad \tilde{q} = -\tilde{\varepsilon} \frac{d\langle q \rangle}{dy}$$

$$\boxed{\frac{\partial}{\partial t} \left\{ \langle v_x \rangle - \left( - \frac{\langle \tilde{q}^2 \rangle}{2\partial \langle q \rangle / \partial y} \right) \right\} = 0}$$

$-\langle \tilde{q}^2 \rangle / 2\partial \langle q \rangle / \partial y \equiv$
Pseudomomentum for QG system

$\rightarrow$ no slip condition of flow + quasi-particle gas

$\rightarrow$ (significant) step toward momentum theorem

i.e. ties flow to wave momentum density

# Zonal Flows I

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$
    *polarization length scale* ↵ ↳ *ion GC* ↳ *electron density*
  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2 \langle \tilde{v}_{rE} \nabla_\perp^2 \tilde{\phi} \rangle \neq 0$ ⬌ 'PV transport'
    ↳ *polarization flux* → What sets cross-phase?
  - If 1 direction of symmetry (or near symmetry):

    $-\rho^2 \langle \tilde{v}_{rE} \nabla_\perp^2 \tilde{\phi} \rangle = -\partial_r \langle \tilde{v}_{rE} \tilde{v}_{\perp E} \rangle$ (Taylor, 1915)

    $-\partial_r \langle \tilde{v}_{rE} \tilde{v}_{\perp E} \rangle$ ➡ Reynolds force ➡ Flow

# Notable by Absence: Three "Usual Suspects"

- "Inverse Cascade"
  - Wave mechanism is essentially linear

    → scale separation often dubious
  - PV transport is sufficient / fundamental
- "Rhines Mechanism"
  - requires very broad dynamic range
  - Waves ⇔ $k_R$ ⇔ forced strong turbulence
  - strong turbulence model
- "Modulational Instability"

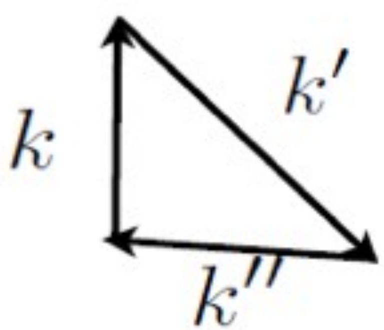
  → see P.D. et al. PPCF'05, CUP'10 for detailed discussion
  - coherent, quasi-coherent wave process
  - useful concept, but not fundamental

Lesson: Formation of zonal bands is generic to the response of a rapidly rotationg fluid to any localized perturbation

# Inverse Cascade/Rhines Mechanism

$k$ : $\omega_k \sim -\beta k_x/k^2$ ; $1/\tau_k$

transfer <=> triad couplings




eddy transfer: $\omega_{MM} < 1/\tau_c$

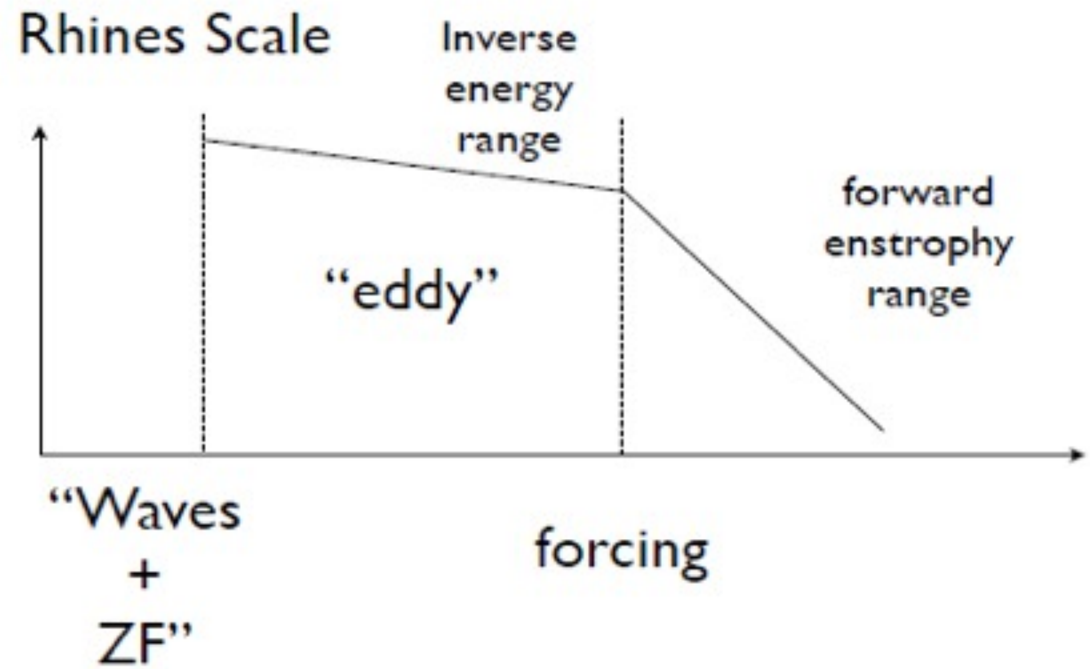
wave transfer: $\omega_{MM} > 1/\tau_c$

cross over: $\omega_{MM} \sim 1/\tau_c$

=> Rhines Scale - emergent characteristic scale for ZF

$$l_R \sim (\tilde{v}/\beta)^{1/2} \sim \epsilon^{1/5}/\beta^{3/5}$$

Contrast: Rhines mechanism vs critical balance




triads: 2 waves + ZF

The crux:

- 3 wave resonance requires 1 wave with $k_x = 0$

- ZF's appear at $k_R$

- coupling maximal at $k_R$

=> $k_R$ Z.F. dominates

$\rightarrow$ Caveat Emptor:

- often said `Zonal Flow Formation $\cong$ Inverse Cascade'

<u>but</u>

- anisotropy crucial $\rightarrow$ $\langle \tilde{V}^2 \rangle$, $\beta$, forcing $\rightarrow$ ZF scale
- numerous instances with: $\Big\langle$ no inverse inertial range; ZF formation $\leftrightarrow$ quasi-coherent

all really needed:

$$\langle \tilde{V}_y \tilde{q} \rangle \rightarrow \text{PV Flux} \rightarrow \langle \tilde{V}_y \tilde{V}_x \rangle \rightarrow \text{Flow}$$

$\rightarrow$ transport and mixing of PV are fundamental elements of dynamics

# Zonal Flows II

- Potential vorticity transport and momentum balance
  - Example: Simplest interesting system → Hasegawa-Wakatani
    - Vorticity: $\frac{d}{dt}\nabla^2\phi = -D_{\|}\nabla_{\|}^2(\phi - n) + D_0\nabla^2\nabla^2\phi$
    - Density: $\frac{dn}{dt} = -D_{\|}\nabla_{\|}^2(\phi - n) + D_0\nabla^2 n$

    $\left\{ \begin{array}{l} D_0 \quad \text{classical, feeble} \\ \Pr = 1 \quad \text{for simplicity} \end{array} \right.$
  - Locally advected PV: $q = n - \nabla\phi^2$
    - PV: charge density $\left\{ \begin{array}{l} n \rightarrow \text{guiding centers} \\ -\nabla\phi^2 \rightarrow \text{polarization} \end{array} \right.$
    - conserved on trajectories in inviscid theory $\boxed{dq/dt=0}$
    - PV conservation → $\left. \begin{array}{l} \text{Freezing-in law} \\ \text{Kelvin's theorem} \end{array} \right\} \rightarrow$ Dynamical constraint

# Zonal Flow II, cont'd

- Potential Enstrophy (P.E.) balance

$d\langle q^2\rangle / dt = 0$ flux dissipation $\langle\ \rangle \rightarrow$ coarse graining

$$\text{LHS} \Rightarrow \frac{d}{dt}\langle \tilde{q}^2\rangle \equiv \partial_t\langle \tilde{q}^2\rangle + \partial_r\langle \tilde{V}_r\tilde{q}^2\rangle + D_0\langle(\nabla\tilde{q})^2\rangle$$

$\text{RHS} \Rightarrow \text{P.E. evolution} -\langle \tilde{V}_r\tilde{q}\rangle\langle q\rangle' \Rightarrow$ P.E. Production by PV mixing / flux

- PV flux: $\langle \tilde{V}_r\tilde{q}\rangle = \langle \tilde{V}_r\tilde{n}\rangle - \langle \tilde{V}_r\nabla^2\tilde{\phi}\rangle$; but : $\langle \tilde{V}_r\nabla^2\tilde{\phi}\rangle = \partial_r\langle \tilde{V}_r\tilde{V}_\theta\rangle$

$\therefore$ P.E. production directly couples driving transport and flow drive

- Fundamental Stationarity Relation for Vorticity flux

$$\langle \tilde{V}_r\nabla^2\tilde{\phi}\rangle = \langle \tilde{V}_r\tilde{n}\rangle + (\delta_t\langle \tilde{q}^2\rangle)/\langle q\rangle'$$

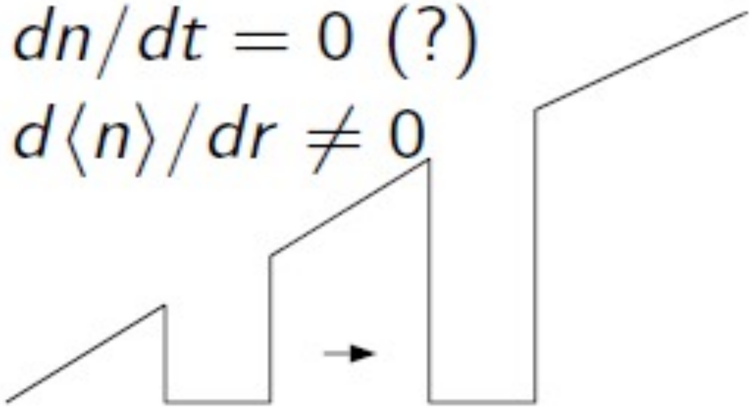
Reynolds force Relaxation Local PE decrement

$\therefore$ Reynolds force locked to driving flux and P.E. decrement; transcends quasilinear theory

# Contrast: Implications of PV Freezing-in Law

$dn/dt = 0$ (?)

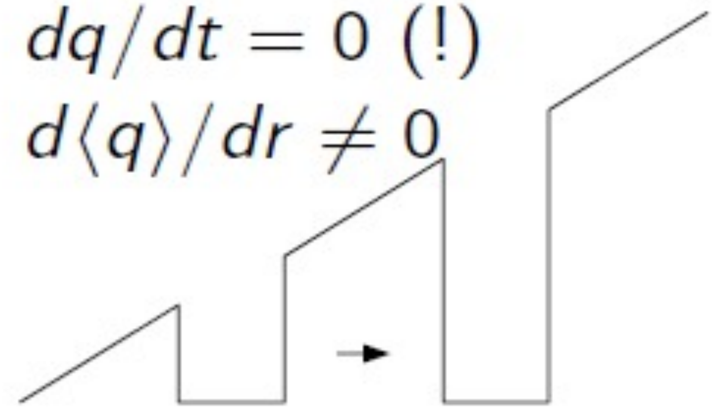
$d\langle n\rangle/dr \neq 0$



$\tilde{n}$ grows $\rightarrow \langle \tilde{V}_r \tilde{n}\rangle \rightarrow$ :-(

$dq/dt = 0$ (!)

$d\langle q\rangle/dr \neq 0$



$\tilde{q}$ grows

$$\rightarrow \begin{cases} \langle \tilde{V}_r \tilde{n}\rangle \rightarrow \ \text{transport} \rightarrow \text{:-(} \\ \langle \tilde{V}_r \nabla^2 \tilde{\phi}\rangle \rightarrow \ \text{flow} \rightarrow \text{:-)} \end{cases}$$

Lesson: Even if $\langle q\rangle \cong \langle n\rangle$, PV conservation must channel free energy into zonal flows!

Key Question: Branching ratio of energy coupled to flow vs transport-inducing fluctuations?

- Combine: $\begin{cases} \text{PE balance} \\ \partial_t \langle V_\theta \rangle = -\langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle - \nu \langle V_\theta \rangle \end{cases}$ yields...
- Charney-Drazin Momentum Theorem

  (1960, et.seq., P.D., et.al. '08, for HW)

$$\Rightarrow \boxed{\partial_t \{ (\overbrace{\text{WAD}}^{\text{Pseudomomentum}}) + \langle V_\theta \rangle \} = -\underbrace{\langle \tilde{V}_r \tilde{n} \rangle}_{\text{driving flux}} - \overbrace{\delta_t \langle \tilde{q}^2 \rangle / \langle q \rangle'}^{\text{local P.E. decrement}} - \underbrace{\nu \langle V_\theta \rangle}_{\text{drag}}}$$

WAD = Wave Activity Density, $\langle \tilde{q}^2 \rangle / \langle q \rangle'$

- pseudomomentum in $\theta$-direction (Andrews, McIntyre '78)
- Generalized Wave Momentum Density

i) momentum of quasi-particle gas of waves, turbulence

ii) consequence of azimuthal/poloidal symmetry

iii) not restricted to linear response, but reduces correctly

- What Does it Mean ? $\rightarrow$ "Non-Acceleration Theorem":

$$\partial_t\{(\mathrm{WAD}) + \langle V_\theta\rangle\} = -\langle \tilde{V}_r \tilde{n}\rangle - \delta_t\langle \tilde{q}^2\rangle / \langle q\rangle' - \nu\langle V_\theta\rangle$$

  - absent $\begin{cases} \langle \tilde{V}_r \tilde{n}\rangle, & \text{driving flux} \\ \delta_t\langle \tilde{q}^2\rangle, & \text{local potential enstrophy decrement} \end{cases}$

    $\rightarrow$cannot $\begin{cases} \text{accelerate} \\ \text{maintain} \end{cases}$ Z.F. with stationary fluctuations!

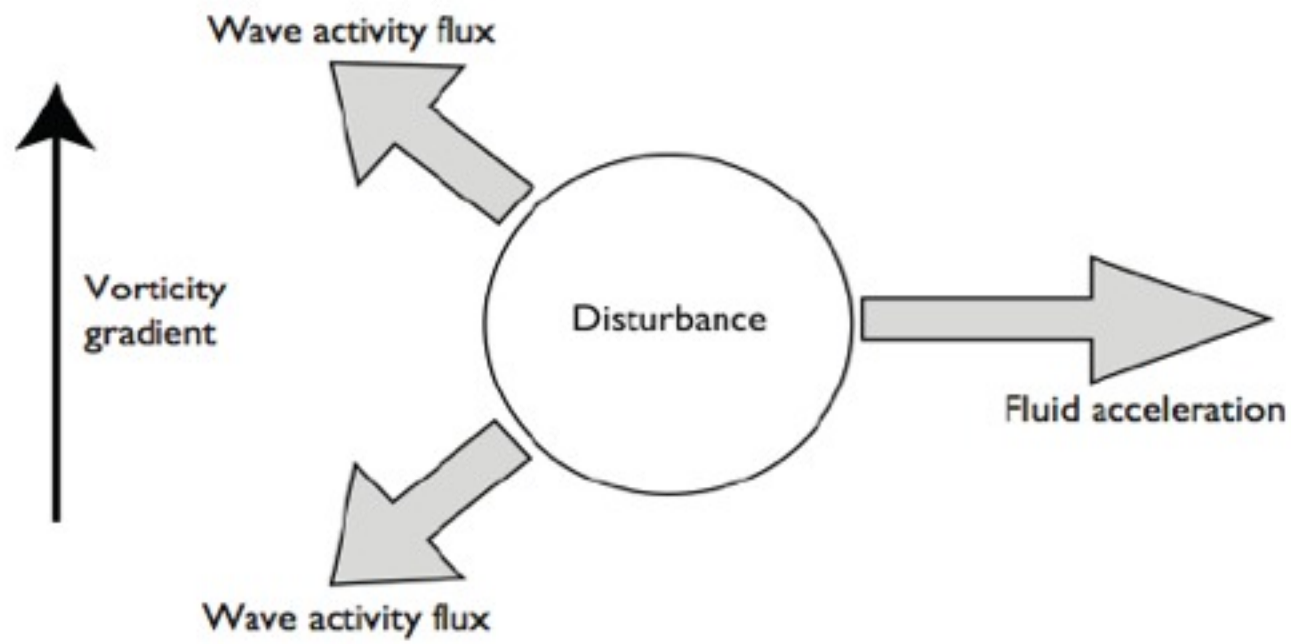
  - Essential physics is PV conservation and translational invariance in $\theta$ $\rightarrow$ freezing quasi-particle gas momentum into flow $\rightarrow$ relative "slippage" required for zonal flow growth

- obvious constraint on models of stationary zonal flows!

  $\leftrightarrow$ need explicit connection to relaxation, dissipation

  N.B. Inhomogeneous dissipation → incomplete homogenization!?

# Aside: H-M




- C-D Theorem for HM

$$\partial_t\{\mathrm{WAD} + \langle V_\theta\rangle\} = \frac{\langle \tilde{f}^2\rangle\tau_c}{\langle q\rangle'} - \frac{1}{\langle q\rangle'}\left\{\partial_r\langle \tilde{V}_r\delta q^2\rangle + \mu\langle(\nabla\delta q)^2\rangle\right\} - \nu\langle V_\theta\rangle$$

- C-D prediction for $\langle V_\theta\rangle$ at stationary state, HM model

$$\langle V_\theta\rangle = \frac{1}{\nu\langle q\rangle'}\left\{\langle \tilde{f}^2\rangle\tau_c - \partial_r\langle \tilde{V}_r\delta q^2\rangle + \mu\langle(\nabla\delta q)^2\rangle\right\}$$

→ Note: Flow direction set by: $\langle q\rangle'$, source, sink distribution

→ Forcing, damping profiles determine shear

→ Potential Enstrophy Transport impact flow structure

# In More Depth: What Really Determines Zonal Flow?

- driving flux: $\langle \tilde{V}_r \tilde{n} \rangle = \Gamma_0 - \Gamma_{col} = \int dr' S_n(r') - \Gamma_{col}$
  - Total flux $\Gamma_0$ fixed by sources, $S_n \rightarrow$ flux driven system
  - Collisional flux in turbulent system, $\Gamma_{col}$ (computed with actual profiles)
  - $\Gamma_o - \Gamma_{col} \rightarrow$ available flux
  - P.E. decrement: $\delta_t \langle \tilde{q}^2 \rangle = \partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle + D_0 \langle (\nabla \tilde{q})^2 \rangle$
    $\rightarrow$ change in roton intensity (PE) changes flow profile
    - roton dissipation
    - P.E. flux, direction increment, according to convergence ($> 0$) or divergence ($< 0$) of pseudomomentum, locally

    So: P.E. transport and "spreading" intrinsically linked to flow structure, dynamics

    Net $\delta$(P.E.) can generate net spin-up

    $\therefore$ Zonal flow dynamics intrinsically "non-local" $\leftrightarrow$ couple to turbulence spreading (fast, meso-scale process)

## Clarifying the Enigma of Collisionless Zonal Flow Saturation

- Flow evolution with: $\nu \to 0$, $S_n \neq 0$ and nearly stationary turbulence

$$\partial_t \langle V_\theta \rangle = -\left( \int dr' S_n(r') - \Gamma_{\mathrm{col}} \right) - \left( \partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle + D_0 \langle (\nabla \tilde{q})^2 \rangle \right) / \langle q \rangle'$$

Possible Outcomes:

- $\langle q \rangle' \to 0$, locally $\to$ shear flow instability (the usual)
  $\leftrightarrow$ limit cycle of burst and recovery, effective viscosity?
  $\to$problematic with magnetic shear
- $\langle \tilde{V}_r \tilde{n} \rangle$ v.s. $\partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle \to$ potential enstrophy transport and inhomogeneous turbulence, with $\tilde{n}/n \sim$ M.L.T
  $\to$ flux drive vs. roton population flux
  $\to$ novel saturation mechanism
- $\langle q \rangle' \to 0$, globally $\to$ homogenized PV state (Rhines, Young, Prandtl, Batchelor)
  $\to$ decouples mean PV, PE evolution
- homogeneous marginality, i.e. $\int dr' S_n(r') = \Gamma_{\mathrm{col}} \leftrightarrow$ ala' stiff core

N.B.: $\langle q \rangle' = 0 \Rightarrow \partial_r \langle n \rangle = \partial_r^2 \langle V_E \rangle = \partial_r \langle \omega_E \rangle$ → particular profile relation !

# Summary of Flow Organization

concept: symmetry

process: PV mixing, transport

constraint released: Enstrophy conservation

players: drift waves

Mean Field: $\Gamma_{PV} = \langle \tilde{v}_r \tilde{q} \rangle$

Global Constraint: Bounding circulation

NL: Pseudomomentum Flux

Outcome: Zonal Flow Formation

Shortcoming: ZF pattern structure and collisionless saturation

## Summary of comparison

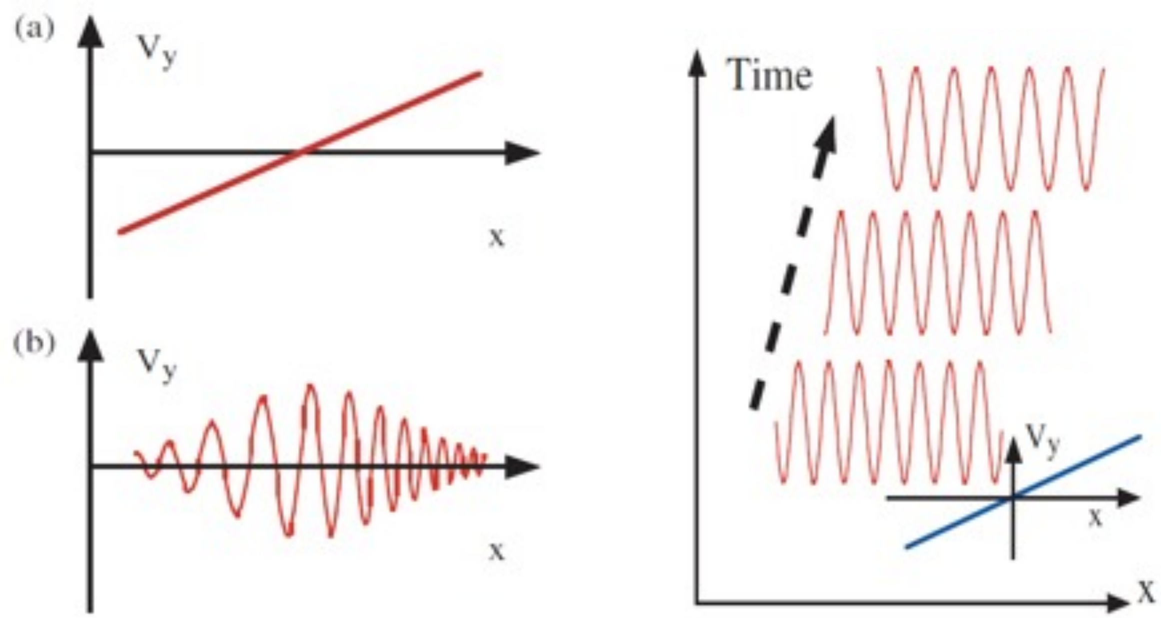
- Many commonalities between magnetic and flow relaxation apparent.

- Common weak point is limitation of mean field theory
  → difficult to grapple with strong NL, non-Gaussian fluctuations.

| | Magnetic (JB) | Flow (GI) |
|---|---|---|
| concept | topology | symmetry |
| process | turbulent reconnection | PV mixing |
| players | tearing modes, Alfven waves | drift wave turbulence |
| mean field | EMF = $\langle \tilde{v} \times \tilde{B} \rangle$ | PV Flux = $\langle \tilde{v}_r \tilde{q} \rangle$ |
| constraint | $\int d^3x \mathbf{A} \cdot \mathbf{B}$ conservation | Potential Enstrophy balance |
| NL | Helicity Density Flux | Pseudomomentum Flux |
| outcome | B-profiles | zonal flow |

# Heuristics of Zonal Flows c.)

- One More Way:
- Consider:
  - –Radially propagating wave packet
  - –Adiabatic shearing field
- $$\frac{d}{dt}k_r = -\frac{\partial}{\partial r}\left(\omega + k_\theta \langle V_{E,ZF} \rangle\right) \Rightarrow \langle k_r^2 \rangle \uparrow$$
- $$\omega_{\vec{k}} = \frac{\omega_*}{1 + k_\perp^2 \rho_s^2} \downarrow$$




- Wave action density $N_k = E(k)/\omega_k$ adiabatic invariant
- $\therefore$ $E(k)\downarrow \Rightarrow$ flow energy decreases, due Reynolds work $\Rightarrow$ flows amplified (cf. energy conservation)
- $\Rightarrow$ Further evidence for universality of zonal flow formation

# Heuristics of Zonal Flows d.) cont'd

- Implications:
  - –ZF's generic to drift wave turbulence in any configuration: electrons tied to flux surfaces, ions not
    - g.c. flux → polarization flux
    - zonal flow
  - –Critical parameters
    - ZF screening (Rosenbluth, Hinton '98)
    - polarization length
    - cross phase → PV mixing
- Observe:
  - –can enhance $e\varphi_{ZF}/T$ at fixed Reynolds drive by reducing shielding, $\rho^2$
  - –typically: $\underbrace{\boxed{\varepsilon/\varepsilon_0}}_{\text{total screening response}} \sim 1+\rho_i^2/\lambda_D^2 + f_t \underbrace{\boxed{\rho_b^2}}_{\text{banana width}}/\lambda_D^2 + f_d \underbrace{\boxed{\delta_d^2}}_{\text{banana tip excursion}}/\lambda_D^2$
  - –Leverage (Watanabe, Sugama) → flexibility of stellerator configuration
    - Multiple populations of trapped particles
    - $\langle E_r \rangle$ dependence (FEC 2010)

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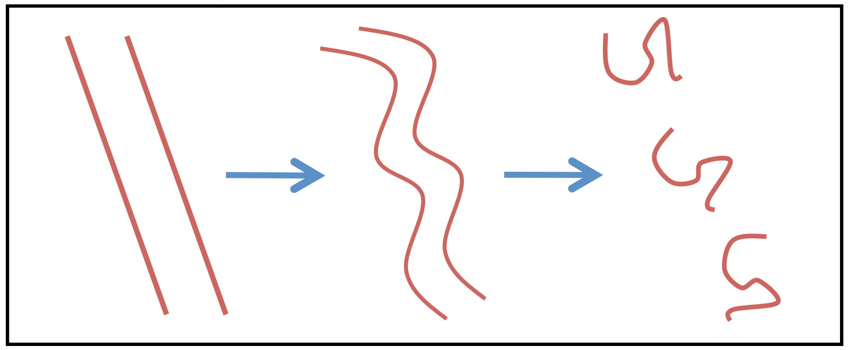
# Heuristics of Zonal Flows d.) cont'd

- Yet more: $\frac{\partial}{\partial t}\langle v_\perp \rangle = -\partial_r \langle \tilde{v}_{rE}\tilde{v}_{\perp E}\rangle - \underbrace{\boxed{\gamma_d \langle v_\perp \rangle}}_{\text{damping}} + \mu \nabla_r^2 \langle v_\perp \rangle$
- Reynolds force opposed by flow damping
- Damping:
  - Tokamak → $\gamma_d \sim \gamma_{ii}$
    - trapped, untrapped friction
    - no Landau damping of (0, 0)
  - Stellerator/3D → $\gamma_d \leftrightarrow NTV$
    - damping tied to non-ambipolarity, also
    - largely unexplored
  - RMP
    - zonal density, potential coupled by RMP field
    - novel damping and structure of feedback loop
- Weak collisionality → nonlinear damping – problematic → tertiary → 'KH' of zonal flow → magnetic shear!? → other mechanisms?





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# Heuristics of Zonal Flows c.) cont'd

## 4) GAMs Happen

- Zonal flows come in 2 flavors/frequencies:
  - –$\omega = 0 \;\Rightarrow$ flow shear layer
  - –GAM $\omega^2 \cong 2c_s^2 / R^2 (1 + k_r^2 \rho_\theta^2) \Rightarrow$ frequency drops toward edge ⇒ stronger shear
    - radial acoustic oscillation
    - couples flow shear layer (0,0) to (1,0) pressure perturbation
    - R ≡ geodesic curvature (configuration)
    - Propagates radially
- GAMs damped by Landau resonance and collisions

  $$\gamma_d \sim \exp[-\omega_{GAM}^2 / (v_{thi} / Rq)^2]$$

  - –q dependence!
  - –edge
- Caveat Emptor: GAMs easier to detect ⇒ looking under lamp post ?!

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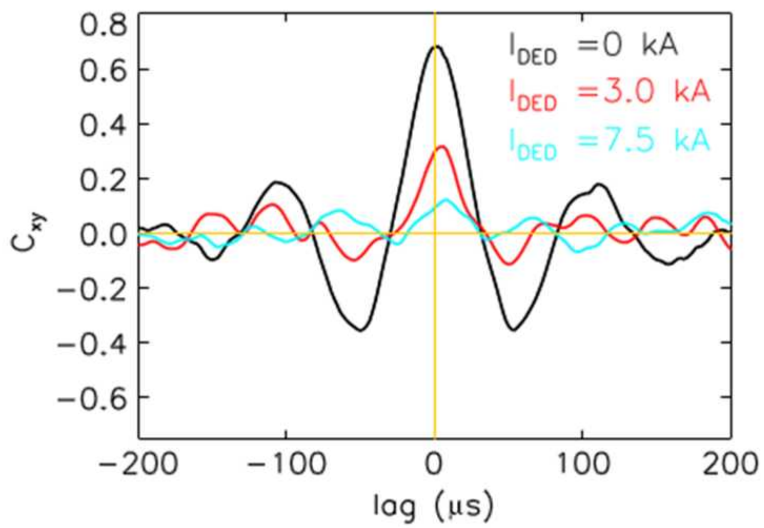
# Progress I: ZF's with RMP (with M. Leconte)

- ITER 'crisis du jour': ELM Mitigation and Control
- Popular approach: RMP
- ? Impact on Confinement?

Y. Xu '11




⇒ RMP causes drop in fluctuation LRC, suggesting reduced Z.F. shearing

⇒ What is "cost-benefit ratio" of RMP?

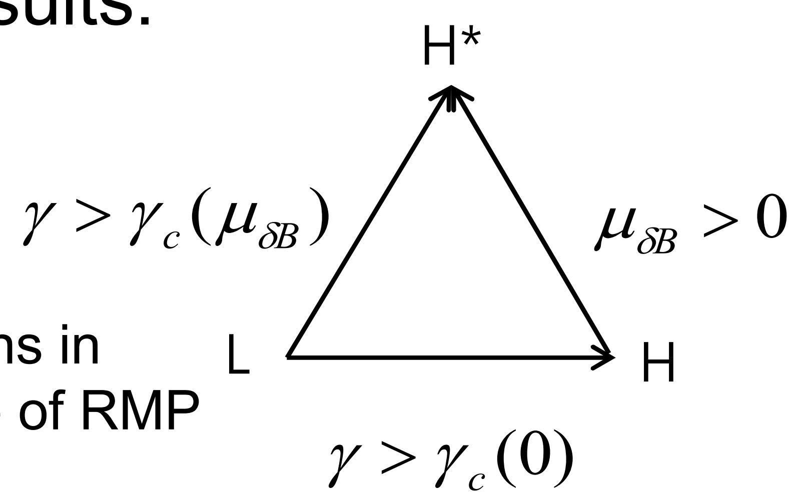
- Physics:
  – in simple H-W model, polarization charge in zonal annulus evolves according:

$$\frac{dQ}{dt} = -\int dA \left[ \left\langle \tilde{v}_x \tilde{\rho}_{pol} \right\rangle + \left( \frac{\delta B_r}{B_0} \right)^2 D_{\parallel} \frac{\partial}{\partial x} \left( \langle \phi \rangle - \langle n \rangle \right) \right] \Bigg|_{r_1}^{r_2}$$

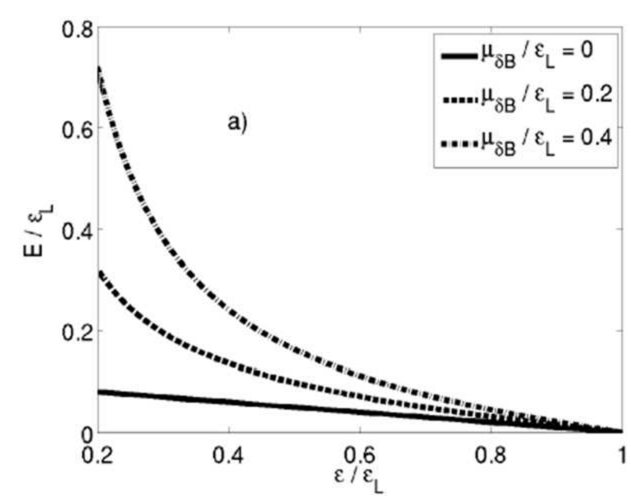
  – Key point: $\delta B_r$ of RMP induces radial electron current → enters charge balance

# Progress I, cont'd

- Implications
  - $\delta B_r$ linearly couples zonal $\hat{\phi}$ and zonal $\hat{n}$
  - Weak RMP $\rightarrow$ correction, strong RMP $\rightarrow$ $\langle E_r \rangle_{ZF} \cong -T_e \partial_r \langle n \rangle / |e|$
- Equations: $$\frac{d}{dt}\delta n_q + D_T q^2 \delta n_q + i b_q (\delta\phi_q - (1-c)\delta n_q) - D_{RMP} q^2 (\delta\phi_q - \delta n_q) = 0$$
$$\frac{d}{dt}\delta\phi_q + \mu\delta\phi_q - a_q(\delta\phi_q - (1-c)\delta n_q) + \frac{D_{RMP}}{\rho_s^2}(\delta\phi_q - \delta n_q) = 0$$
- Results:




Transitions in presence of RMP




$E_{ZF}/\mathcal{E}_L$ vs $\mathcal{E}/\mathcal{E}_L$ for various RMP coupling strengths

## Progress II : β-plane MHD (with S.M. Tobias, D.W. Hughes)

### Model

- Thin layer of shallow magneto fluid, i.e. solar tachocline
- β-plane MHD ~ 2D MHD + β-offset   i.e. solar tachocline

$$\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla \nabla^2 A + \tilde{f}$$

$$\partial_t A + \nabla \phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \qquad \vec{B}_0 = B_0 \hat{x}$$

- Linear waves: Rossby – Alfven $\omega^2 + \omega\beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0$ (R. Hide)
- cf P.D., et al; Tachocline volume, CUP (2007)

  S. Tobias, et al: ApJ (2007)

# Progress II, cont'd

## Observation re: What happens?

- Turbulence → stretch field → $\langle \tilde{B}^2 \rangle >> B_0^2$ i.e. $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m$
  (ala Zeldovich)
- Cascades : - forward or inverse?
  - MHD or Rossby dynamics dominant !?
- PV transport: $\frac{dQ}{dt} = -\int dA \langle \tilde{v}\tilde{q} \rangle$ ⟶ net change in charge content due PV/polarization charge flux

Now $\frac{dQ}{dt} = -\int dA \left[ \langle \tilde{v}\tilde{q} \rangle - \langle \tilde{B}_r \tilde{J}_\parallel \rangle \right] = -\int dA \partial_x \left\{ \langle \tilde{v}_x \tilde{v}_y \rangle - \langle \tilde{B}_x \tilde{B}_y \rangle \right\}$ ⟶ Reynolds mis-match

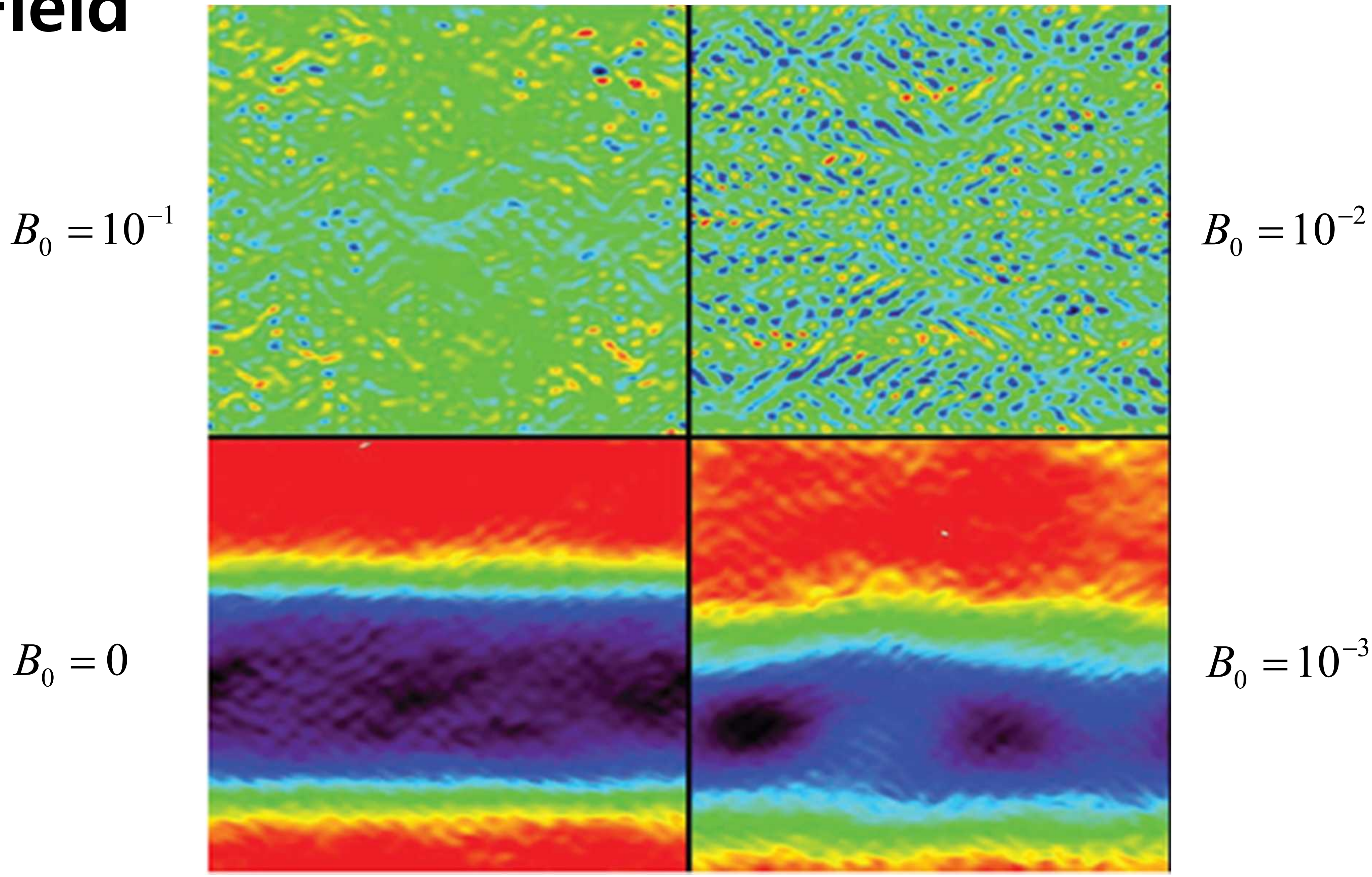
PV flux ↑ current along tilted lines ↑

⟶ vanishes for Alfvenized state

Taylor: $\langle \tilde{B}_x \tilde{J}_\parallel \rangle = -\partial_x \langle \tilde{B}_x \tilde{B}_y \rangle$

# Progress II, cont'd

- **With Field**



# Progress II, cont'd

- Control Parameters for $\vec{\tilde{B}}$ enter Z.F. dynamics

  Like RMP, Ohm's law regulates Z.F.

- Recall
  - $\langle \tilde{v}^2 \rangle$ vs $\langle \tilde{B}^2 \rangle$
  - $\langle \tilde{B}^2 \rangle \sim B_0^2 R_m$ $\longrightarrow$ origin of $B_0^2 / \eta$ scaling !?

- Further study $\rightarrow$ differentiate between :
  - cross phase in $\langle \tilde{v}_r \tilde{q} \rangle$ and O.R. vs J.C.M
  - orientation : $\vec{B} \parallel \vec{V}$ vs $\vec{B} \perp \vec{V}$
  - spectral evolution