For linear theory of CDIA;

$$\nabla^2\hat{\phi} = -4\pi n_0|e|\left(\frac{\hat{n}_i}{n_0} - \frac{\hat{n}_e}{n_0}\right)$$

$$\hat{n}_i/n_0 = \frac{k^2 c_s^2}{\omega^2}\frac{|e|\phi}{T}$$

$$\frac{\hat{n}_e}{n_0} = \frac{|e|\hat{\phi}}{T}\left[1 - i\Gamma(k)\right]$$

n $\Gamma(k)$,

$$\frac{\partial\tilde{f}}{\partial t} + v\frac{\partial\tilde{f}}{\partial x} = -\frac{|e|}{m_e}\tilde{E}\frac{\partial\langle f\rangle}{\partial v}$$

$$\hat{f} = \frac{|e|\hat{\phi}}{T}\langle f\rangle + g$$

$$\frac{\partial\tilde{g}}{\partial t} + v\frac{\partial\tilde{g}}{\partial x} = -v\frac{\partial}{\partial x}\left(\frac{|e|\tilde{\phi}\langle f\rangle}{T}\right) + \frac{|e|}{m_e}\frac{\partial\tilde{\phi}}{\partial x}\frac{\partial\langle f\rangle}{\partial v}$$

$$- \frac{\partial}{\partial t}\left(\frac{|e|\tilde{\phi}\langle f\rangle}{T_e}\right)$$

$$= v\frac{\partial\tilde{\phi}}{\partial x}\frac{|e|}{T}\langle f\rangle + \frac{|e|}{\cancel{m_e}}\frac{\partial\tilde{\phi}}{\partial x}\left(-\frac{(v-\bar{v})}{T/\cancel{m_e}}\right)\langle f\rangle - \frac{\partial}{\partial t}\frac{|e|\tilde{\phi}\langle f\rangle}{T_e}$$

$$= -\frac{\partial}{\partial t}\frac{|e|\tilde{\phi}}{T}\langle f\rangle + \bar{v}\frac{\partial}{\partial x}\frac{|e|\tilde{\phi}}{T}\langle f\rangle$$

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$$\Rightarrow g_k = \frac{i(\omega - k\bar{V})}{-i(\omega - kV)} \frac{|e|\hat{\phi}_k}{T} \langle f \rangle$$

$$= -\left(\frac{\omega - k\bar{V}}{\omega - kV}\right) \frac{|e|\hat{\phi}_k}{T} \langle f \rangle$$

$$\omega_k^{2\,(0)} = \frac{k^2 c_s^2}{1 + k^2 \lambda_D^2}$$

$$-i\, r(k) = \int dV + \frac{(\omega - k\bar{V})}{(\omega - kV)} \langle f \rangle$$

$$\bar{F} = \frac{1}{\sqrt{\pi}} \exp\left[\frac{-(\omega/k - \bar{V})^2}{V_{Th}^2}\right]$$

$$= -(\omega - k\bar{V}) \frac{(+i\pi)}{|k| V_{Th}} \bar{F}\Big|_{\omega/kV_{Th}}$$

$$1 + k^2 \lambda_D^2 = \frac{k^2 c_s^2}{\omega^2} + \frac{(\omega - k\bar{V})(i\pi)}{|k| V_{Th}} \bar{F}\Big|_{\omega/kV_{Th}}$$

$$\omega \to \omega + \delta\omega$$

$$0 = -\frac{2\delta\omega}{\omega} + \frac{(\omega - k\bar{V})(-i\pi)}{|k| V_{Th}} \bar{F}\Big|_{\omega/kV_{Th}}$$

$$\frac{\delta\omega}{\omega} = -\frac{i\pi}{2} \frac{(\omega - k\bar{V})}{|k| V_{Th}} \bar{F}\Big|_{\omega/kV_{Th}}$$

$\delta\omega \Rightarrow i\gamma_k$ growth rate

(a) $$\boxed{\gamma_k \sim \frac{+\pi}{2} \omega_k \frac{(\omega - k\bar{V})}{|k| V_{Th}} \bar{F}\Big|_{\omega/kV_{Th}}}$$ $\Rightarrow$ $\gamma > 0$ for $\bar{V}_{ii} > c_s$

$\Rightarrow$ critical velocity

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for $\langle f \rangle$ evolution,

$$\frac{\partial \langle f \rangle}{\partial t} = +\frac{\partial}{\partial v} \sum_k \frac{+|e|}{m_e} \tilde{E}_{-k}\, \tilde{g}_k$$

$$= +\frac{\partial}{\partial v} \sum_k \frac{|e|}{m_e} \tilde{E}_{-k} \left(\frac{-(\omega - k\bar{v})}{(\omega - kv)} \frac{|e|\tilde{\phi}_k}{T} \langle f \rangle \right)$$

$$= \frac{\partial}{\partial v} \sum_k \frac{T_e |e|}{m_e} \frac{+ik\,\tilde{\phi}_k^*}{T_e} \left(-(\omega - kv)\, i\pi\delta(\omega - kv) \right) \frac{|e|\tilde{\phi}_k}{T} \langle f \rangle$$

$$= \frac{\partial}{\partial v} \sum_k (-v_{th}^2) \left| \frac{|e|\tilde{\phi}_k}{T} \right|^2 k(\omega - k\bar{v})\, \pi\delta(\omega - kv) \langle f \rangle$$

$$\boxed{\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} \sum_k (-v_{th}^2) \left| \frac{|e|\tilde{\phi}_k}{T} \right|^2 k(\omega - k\bar{v})\, \pi\delta(\omega - kv) \langle f \rangle} \quad (8)$$

→ mean evolution

Note:

- really only assumed $\langle f \rangle = \langle f\left((v-\bar{v})^2 / 2v_{th}^2\right) \rangle$

so $\displaystyle \frac{\partial \langle f \rangle}{\partial v} = \left(\frac{v - \bar{v}}{v_{th}^2} \right) \langle f \rangle$

and

$\langle f \rangle' = -\langle f \rangle$

→ minimal assumption on structure

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- can write as $\bar{v}$ evolution

$$\bar{v} = \int dv\, v \langle f \rangle \Big/ \int dv \langle f \rangle$$

$$\Rightarrow \boxed{\frac{\partial \bar{v}}{\partial t} = + \int dv \sum_k v_{Th}^2 \left| \frac{e\hat{\phi}_k}{T} \right|^2 k^2 \left(\frac{\omega}{k} - \bar{v} \right) \pi \delta(\omega - kv) \langle f \rangle}$$

$\omega/k < \bar{v} \Rightarrow \partial \bar{v}/\partial t < 0 \rightarrow$ slows down

$> \bar{v} \rightarrow \partial \bar{v}/\partial t > 0$

- Remains to determine fluctuation intensity level

Generically can write:

$$\frac{\partial}{\partial t} \mathcal{E}_k^{\omega} = \overset{(a)}{\gamma_k \mathcal{E}_k^{\omega}} - \overset{(b)}{\left(\sum_{k'} \omega_k\, C_1(k, k') \frac{\mathcal{E}_{k'}^{\omega}}{nT} \right)} \mathcal{E}_k^{\omega}$$

$$- \overset{(c)}{\left(\sum_{k', k''} \omega_k\, C_2(k, k', k'') \frac{\mathcal{E}_{k'}^{\omega}}{nT} \frac{\mathcal{E}_{k''}^{\omega}}{nT} \right)} \mathcal{E}_k^{\omega}$$

spectral equation constituents:

(a) - linear growth

(b) - quadratic nonlinearity $\rightarrow$ 3 wave coupling / NL ion-wave interaction

(c) - cubic NL $\rightarrow$ wave coupling

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Now, for ion-acoustic wave:

- 3 wave coupling effects negligible
  ⇒ can't satisfy resonance
- NL wave-particle effects weak ⇒ intrinsically
⇒ consider 4 wave process

$$\boxed{\frac{\partial \varepsilon_k^\omega}{\partial t} = \left[\gamma_k - \omega_k \, B(\omega, k) \left(\frac{\varepsilon_k^\omega}{nT} \right)^2 \right] \varepsilon_k^\omega} \quad (5)$$

- 'cartoon' NL saturation equation

Now, ①-⑤ ⇒ {coupled, ⓐ-stationary micro-macro system

⇒ describe anomalous resistivity dynamics and its effect on reconnection

⇒ coupled solution corresponds to solution of the problem

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(1)
$$\begin{cases} n m_e \nu_{eff}(B,\Delta)\,\bar{V} = \sum_k 2\gamma_k^e \dfrac{k}{\omega_k} \mathcal{E}_k^W \\ \Delta^2 = \dfrac{L}{V_A}\left(M + \dfrac{c^2}{\omega_{pe}^2}\nu_{eff}\right), \quad \bar{V} = cB/4\pi n q \Delta \end{cases}$$

(2)
$$\gamma_k^e = -\frac{\pi}{2}\,\omega_k\,\frac{(\omega - k\bar{V})}{|k|\,v_{Th}}\,\bar{F}\Big|_{\omega/k v_{Th}}$$

(4)
$$\frac{\partial \bar{V}}{\partial t} = \int dv\left(\sum_k v_{Th}^2\left|\frac{e\tilde{\phi}_k}{T}\right|^2 k^2\left(\frac{\omega}{k} - \bar{V}\right)\pi\delta(\omega - kv)\langle F\rangle\right)$$

(5)
$$\frac{\partial \mathcal{E}_k^W}{\partial t} = \left[\gamma_k - \omega_k\,B(\omega,k)\left(\frac{\mathcal{E}_k^\omega}{nT}\right)^2\right]\mathcal{E}_k^W$$

Now, stationarity $\Rightarrow$

$$\mathcal{E}_k^W = nT\left(\gamma_k/\omega_k B\right)^{1/2}$$

$$\gamma_k = +\frac{\pi}{2}\frac{(\bar{V} - C_s)}{|k v_{Th}|}\,k\omega_k\,\bar{F}\Big|_{\omega/k v_{Th}}.$$

so, for scalings:

$$\nu_{eff} = \frac{1}{nm\bar{V}} \sum_k 2\gamma_k^e \frac{k}{\omega_k} \mathcal{E}_k^{\omega}$$

$$\sim \frac{1}{nm\bar{V}} \frac{(\bar{V}-c_s)}{|k|v_{Th}} k\cancel{\omega_k} \bar{f}\Big|_{\frac{\omega_k}{kv_{Th}}} \frac{k}{\cancel{\omega_k}}(nT)\left(\frac{\gamma_k}{\omega_k B}\right)$$

$$\sim \frac{(1-c_s/\bar{V})}{nm} \frac{k^2 nT}{|k|v_{Th}} \left(\frac{\gamma_k}{\omega_k B}\right) \bar{f}\Big|_{\frac{\omega_k}{kv_{Th}}}$$

$$\sim (1-c_s/\bar{V})\,\bar{f}\Big|_{\omega/|k|v_{Th}} \left(\frac{k^2}{|k|} v_{Th}\right)\left(\frac{\gamma_k}{\omega_k B}\right)$$

$$\sim (1-c_s/\cancel{\bar{V}})\left(\frac{k\omega_k}{|k v_{Th}|}\frac{(\bar{V}-c_s)}{\cancel{\omega_k}B}\bar{f}\right)^{1/2} \bar{f}\,\frac{k^2 v_{Th}}{|k|}$$

$$\sim \left[(\bar{V}-c_s)k\right]^{3/2} \bar{f}^{3/2} \frac{k(v_{Th}/\bar{V})}{|k|\,|k v_{Th}|^{1/2}}$$

$$\boxed{\nu_{eff} \simeq \frac{\left[(\bar{V}-c_s)k\right]^{3/2}}{|k v_{Th}|^{1/2}} \frac{v_{Th}}{\bar{V}} \bar{f}^{3/2}\left(\frac{k}{|k|}\right)}$$

[Turbulent collision frequency

$\gamma \uparrow$ with $\bar{V} > c_s$

so have:

$$\boxed{\begin{aligned} \Delta^2 &= \frac{b}{V_A}\left(\Lambda + \frac{c^2}{\omega_{pe}^2}\nu_{eff}\right) \\ \bar{V} &= cB_0/4\pi n_0 q\Delta \\ \nu_{eff} &= \frac{\left[(\bar{V}-c_s)k\right]^{3/2}}{|k v_{Th}|^{1/2}}\frac{v_{Th}}{\bar{V}}\bar{F}^{3/2}\left(\frac{k}{|k|}\right) \end{aligned}}$$

with:

$$\boxed{\begin{aligned} \frac{\gamma_k}{\omega_k} &= \frac{\pi}{2}\left(\bar{V}-c_s\right)\frac{k}{|k|v_{Th}}\bar{F} \\ \bar{F} &= \frac{1}{\sqrt{\pi}}\exp\left[-\left(\omega/k-\bar{V}\right)^2/2v_{Th}^2\right] \end{aligned}}$$

→ characterize micro-macro coupling with anomalous resistivity

→ now can envision situation
- finite current, $\Delta \sim (b\Lambda/V_A)^{1/2}$
  $\nu_{eff} = 0$

so if:

- decrease Λ ⇒ Δ decreases
- Δ decreases ⇒ $\bar{V}$ increases

- $\bar{V}$ increases $\Rightarrow \gamma_k > 0$
- $\gamma_k > 0 \Rightarrow \begin{cases} \mathcal{E}^{w}_{k} > 0 \\ \nu_{eff} > 0 \end{cases}$
- $\nu_{eff} > 0 \Rightarrow \begin{cases} \Delta \text{ increases} \\ \bar{V} \text{ decreases} \end{cases}$

$\Rightarrow$ decreasing ν so Δ decreases triggers feedback so Δ increases $\Rightarrow$ self-regulation / $\oplus$ feedback

$\therefore$ in this model, can expect:

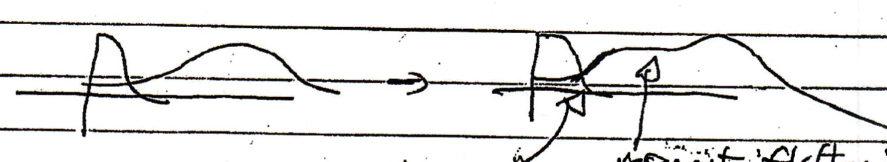
- at low $\nu_{collisions}$, so $\bar{V}\big|_c \sim cB_0/4\pi n_0 e \Delta_c$
$\sim C_s$

$\Rightarrow$ CDIA "hovers" near marginal stability

- for stronger drive (i.e. $B_0 \uparrow$)

$\Rightarrow$ ion interaction important
$\Rightarrow$ strong ion distortion possibly significant
$\Rightarrow$ granulation formation important
$\Rightarrow$ distortion of electron distribution function need be considered

i.e.



⇒ useful extensions :

- 1D avalanche model → avalanching ⇒ jitter effects on Δ, $\overline{V}$

- nonlinear noise effects via fluctuations

- 2D, 3D ⇒ wave radiation, esp. wave momentum flux ⊥ layer.

* - granulation effects ⇒ strong distortion hole (cf. later in course)

Comment:

This simple problem is surprisingly poorly understood. Excellent example of:

→ micro-macro feedback

→ self-regulation

→ marginal stability ...