

## Simple Ideas in Non-Ideal MHD I

→ Freezing-in law, follow from induction

$$\partial_t \underline{B} = \underline{V} \times (\underline{V} \times \underline{B})$$

→ Now, allow breaking of freezing-in

⇒ resistive, not ideal, MHD

$$\partial_t \underline{B} = \underline{V} \times (\underline{V} \times \underline{B}) + \underbrace{\mu \underline{\nabla}^2 \underline{B}}_{\uparrow}$$

$$\underline{J} = \underline{V} \times \underline{B} \frac{c}{4\pi}$$

+  $\infty$  # variants

|                |                   |
|----------------|-------------------|
| - Hall MHD     | - $c/\omega_{pi}$ |
| - EMHD         | - $c/\omega_{pe}$ |
| - Drift-Alfven | - $\rho_i$        |
| ...            |                   |

→ observe:

- ideal → resistive MHD is a singular perturbation

induction from 1<sup>st</sup> order PDE → 2<sup>nd</sup> order

⇒ diffusion + induction

Analogy is Euler  $\rightarrow$  Navier-Stokes

$$\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \underline{v}$$

drag, wakes.

(irreversible)

(wet water)  $\rightarrow$  RPF

from

$$\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} = -\frac{1}{\rho} \nabla P$$

no drag, wakes

(reversible)

(dry water)  $\rightarrow$  RPF

$\rightarrow$  Linear Phenomena?

- non-dissipative  $\left\{ \begin{array}{l} \text{All Even} \\ \text{slow} \\ \text{Fast} \end{array} \right.$  wave

+

- dissipative - diffusion

$\rightarrow$  How characteristic? "How ideal?"

$$\partial_t \underline{B} = \nabla \times \underline{v} \times \underline{B} + \nabla^2 \underline{B}$$



For Fluid:  $Re \sim \frac{U \cdot D}{\nu} \sim \frac{VL}{\nu}$

Reynolds #

(scale dependence)

MHD:  $R_m \sim \frac{U \times V \times B}{\mu D^2}$   $\sim \frac{VL}{\eta}$

Magnetic  
Reynolds #

( $\eta \approx \nu$  in general)

'Interesting' non-ideal flow has high  $R_m$  (i.e.  $> 10^2$ )  $\Rightarrow$  clear separation of dynamical and diffusive scales

$\Rightarrow$  violation of freezing in occurs on small scales, at large  $R_m$ .

But, what of stationary problems?  
(i.e. instability growth from equilibrium)

Consider comparison of characteristic time scales:

$$\frac{1}{\tau_A} \sim \frac{v_A}{L}$$

$$\delta$$

$$(\text{Alfven time})^{-1}$$

$$\frac{1}{\tau_R} \sim \frac{\eta}{L^2}$$

$$(\text{Resistive time})^{-1}$$

$$\frac{1}{S} \sim \frac{\eta}{L^2} \frac{L}{v_A} \sim \frac{\eta}{L v_A}$$

$$\downarrow$$

Lundquist #  $(\sim 1/R_m, \text{ with } v \sim v_A)$

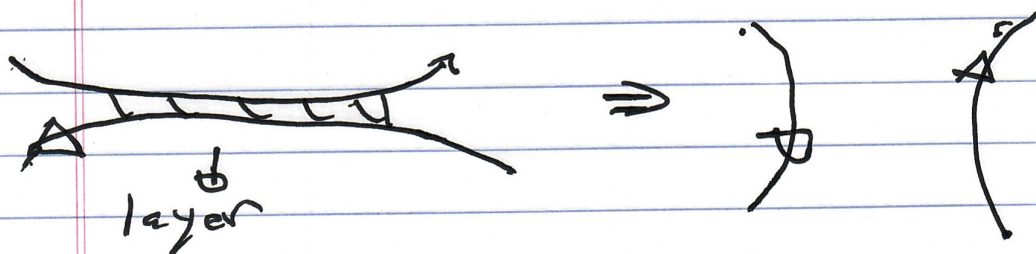
Need  $S > 10^3$  (roughly) for  
dynamics of interest

Both  $1/S$  and  $R_m$  favor small scales  
as loc of field-fluid decoupling.



→ Non-Ideal Behavior ?

- Magnetic Reconnection



Change in magnetic topology/linkage associated with

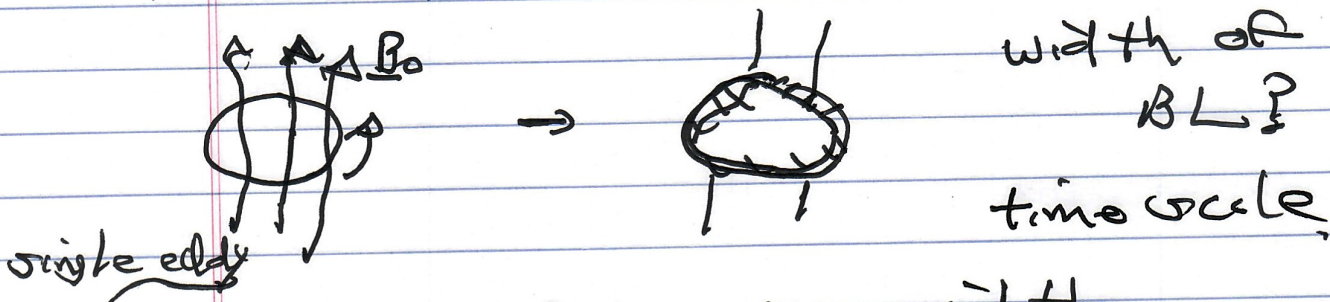
- resistive dissipation of current sheets

- wave radiation

→ wave-particle momentum exchange

Issue: Rate → how does reconnection rate scale with  $\left\{ \begin{array}{l} S \\ R_m \end{array} \right.$  ?

- Flux Expulsion



Issues: Rate, BL width  $B_0$  to prevent

## - Magnetic Helicity and Relaxation

→  $\mathcal{H} = \int d^3x \underline{A} \cdot \underline{B}$  - B-field linkage

→ Taylor Relaxation, and mean field theory → hyper-resistivity

↓

constraint on magnetic dissipation

## - MHD Turbulence ( $\underline{D} \cdot \underline{v} = 0$ )

→ Alfvén Waves + Eddys.

→  $\kappa$ -I + G-S models

## - Intro to Dynamics

→ how make an ordered field  $\hat{n}$  <sup>amplitude</sup> turbulent system?

→ symmetry ↓ - STF



Ex.

→ calculating  $\langle \tilde{V} \times \tilde{B} \rangle$  - mean EMF

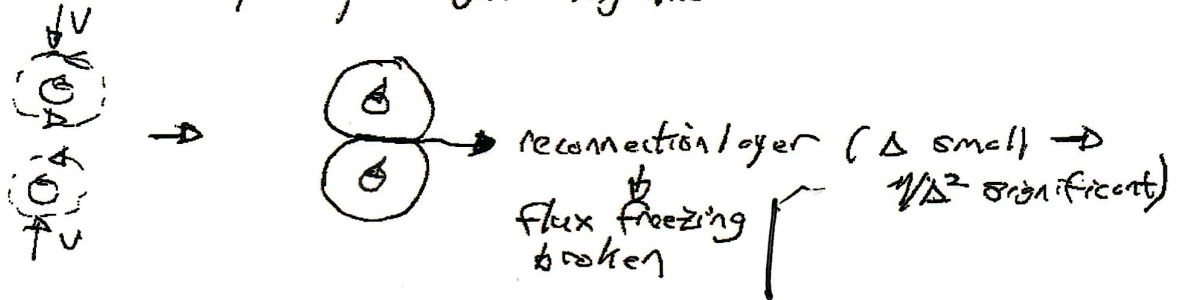
First: Basics of Reconnection:

Sweet - Parker Problem

# → Breakdown of Flux Freezing - Magnetic Reconnection?

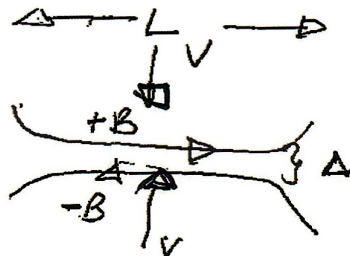
Simple Example: Sweet - Parker Problem  
(re-visit later)

→ consider two cylinders of plasma, carrying current  $\perp$  plane, brought together



⇒ consider layer

[ 2 plasma slabs  
brought together  
at  $V$



current sheet?

$\Delta \ll L$   
What Happens?  
Stationary  
Solution  
Possible?

$$\nabla \cdot \underline{v} = 0$$

$$\frac{d\underline{B}}{dt} = \underline{B} \cdot \nabla \underline{v} + \eta \nabla^2 \underline{B}$$

$$\underline{S} = \begin{pmatrix} 0 & 0 \\ 0 & -v \frac{dy}{dx} \end{pmatrix}$$

rate of strain tensor

→ singularity

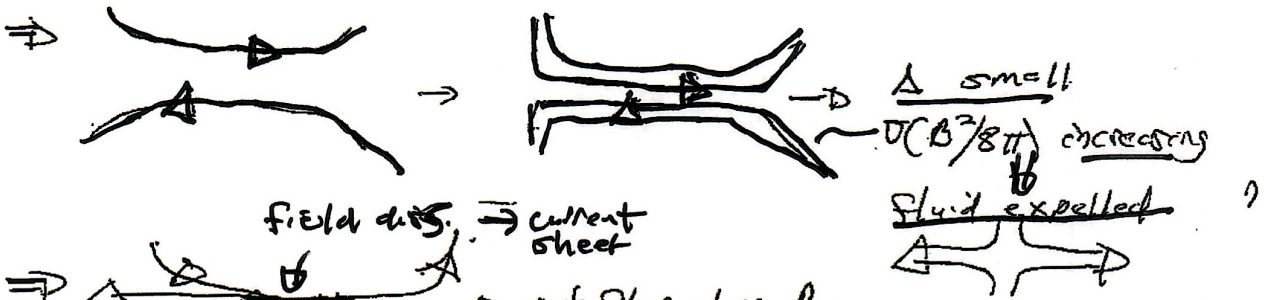
tip-off of small scale generation in  $\underline{B}$   
⇒ resistive diffusion, breaking of freezing in.



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i.e. for stationary solution;

$$\underbrace{- \frac{\mathbf{B} \cdot \nabla \mathbf{V}}{\eta} = \nabla^2 \mathbf{B}}_{\text{}} \quad , \quad \nabla \cdot \mathbf{V} = 0$$

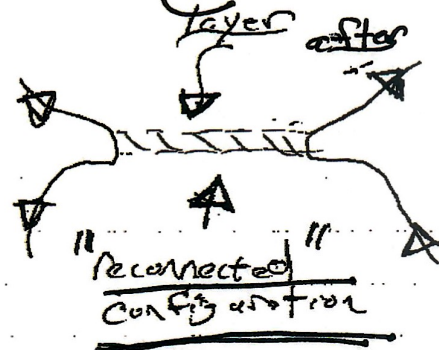
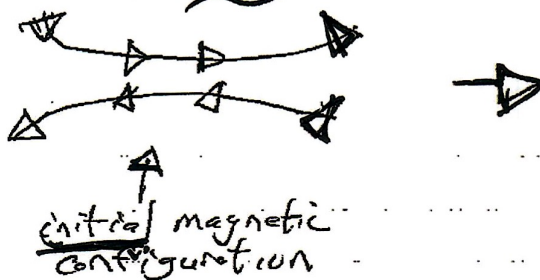


(large resistive dissipation)

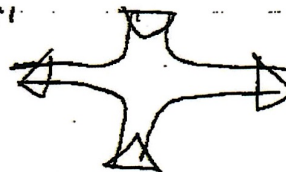
expect opposed fields "annihilate"

N.B. A particular  $V$  value is required for stationarity.

N.B. → why "reconnection"?



→ Flow is "stagnation point"



also shock

→ How Calculate? → Match In-Flow → Out-Flow  
 (S-P is a great Back-of-Envelope →)  
 Conserved: ① - mass ( $\underline{U} \cdot \underline{V} = 0$ )

② - momentum in  $\hat{x}$  direction (symmetry)

③ - energy balance →  
 rate of field 'delivery' to reconnection region  
 MUST BALANCE  
 rate of Ohmic dissipation  $E \cdot J \sim 4J^2$

①

extent in  $\hat{x}$  extent in  $\hat{y}$  mass ~~length~~  $\Delta$

$\rho_0 V_L = \rho_0 V_0 \Delta$

inflow outflow

$V_L = V_0 \Delta$

$\underline{D} \cdot \underline{V} = C$

$V = V_{out} \Delta / L$

②

$$\rho_0 \left( \frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot \nabla \underline{V} \right) = - \nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{B \cdot \nabla B}{4\pi}$$

$$\underline{V} \cdot \nabla \underline{V} = - \nabla \left( \frac{V^2}{2} \right) + \underline{V} \times \underline{\omega}$$

symmetry:  $0 = \nabla \left( p + \frac{B^2}{8\pi} + \frac{\rho_0 V^2}{2} \right)$

modified  
Bernoulli  
Eqn.

$\underline{a} \rightarrow \underline{b}$

$\underline{a} \rightarrow \underline{V} = 0, \quad B \text{ finite}$

$\underline{b} \rightarrow \underline{V} = V_{out}, \quad B \rightarrow 0$

$\left( \frac{B^2}{8\pi} \ll p \right)$



So  $\rho + \frac{B^2}{8\pi} + \frac{\rho_0 V^2}{2} = \text{const.}$

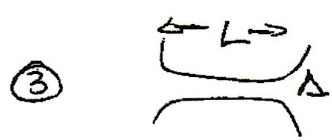
$$\rho + \frac{B^2}{8\pi} = \rho + \frac{\rho_0 V_{out}^2}{2}$$

$$V_{out}^2 = \frac{B^2}{4\pi\rho_0} = V_A^2$$

→ Alven speed

$$\begin{aligned} V_{out} &= V_A \\ V &= V_A \frac{\Delta}{L} \end{aligned}$$

specifies "speed"  $V$ , in terms  $\Delta$ .



Energy balance

$$\Rightarrow \left[ \begin{array}{l} \text{Rate of Magnetic} \\ \text{Energy Inflow} \end{array} \right] = \left[ \begin{array}{l} \text{Rate of Ohmic} \\ \text{Dissipation, net} \end{array} \right]$$

$$P_{oh} = \frac{J^2 \Delta L}{\underbrace{\frac{1}{\sigma}}_{\text{Area}}} \quad \text{so} \quad \dot{E}_{oh} = \frac{J^2}{\sigma} L \Delta$$

$$= \left( \frac{c}{2\pi} \right)^2 \frac{B^2 L \Delta}{\Delta^2 \sigma}$$

$$\nabla \times B = \frac{4\pi}{c} J$$

$$2B = \frac{4\pi J \Delta}{c}$$

$$P_{in} = 2 \left( \frac{B^2}{8\pi} \right) VL = \dot{E}_{in}$$

balance  $\Rightarrow \cancel{2} \left( \frac{\cancel{B}^2}{\cancel{8\pi}} \right) V \cancel{L} = \frac{c^2}{4\pi} \frac{\cancel{B}^2}{\cancel{\Delta}^2} \cancel{L} \Delta$

$$\frac{c^2}{4\pi\sigma} \equiv \eta \left( \sim \frac{L^2}{\tau} \right)$$

$$V = \left( \frac{c^2}{4\pi\sigma} \right) / \Delta \sim \frac{\eta}{\Delta}$$

$$V_A \frac{\Delta}{L} \sim \frac{\eta}{\Delta}$$

$$\frac{V}{L} \sim \left(\frac{\mu}{L^2}\right)^{1/2} \frac{V_A^{3/2}}{\sqrt{\mu}} \sqrt{L}$$

$$\sim \left(\frac{\mu}{L^2}\right)^{1/2} \frac{V_A}{L}$$

$$\sim \mu^{-1/2} \frac{V_A}{L}$$

$$V = V_A \Delta / L$$

$$V \sim \frac{V_A \mu^{1/2}}{L^{1/2}}$$

$$1/\tau \sim \frac{V}{L}$$

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$$V = \mu / \Delta$$

$$\frac{\Delta}{L} = \left(\frac{\mu L}{L^2 V_A}\right)^{1/2} = \left(\frac{\mu}{R_m}\right)^{1/2}$$

$$R_m = \frac{VL}{\mu} \equiv \text{Magnetic Reynolds \#}$$

(here with  $V = V_A$ )

and

$$V = V_A / \sqrt{R_m}$$

⇒ Punch Line: ① - layer is thin

(for large  $R_m$ ) - speed is faster than  $\mu/L$ , slower than  $V_A$

$$\frac{\Delta}{L} \sim 1/\sqrt{R_m}$$

$$V \sim V_A / \sqrt{R_m}$$

② Flow pattern is also stagnation ⇒  $\left\{ \begin{array}{l} \text{ejection from} \\ \text{reconnection layer} \\ \text{at } V_A \end{array} \right.$

Moral of this story:  $\frac{\tau}{T_R} \sim \frac{V}{L} \sim \frac{(V_A)^{1/2} \mu^{1/2}}{L^{3/2}}$

→ Freezing-in violated. when flows bring opposing  $\underline{B}$  into contact

⇒

→ generates singularities ⇒ thin current layers, which alter critical magnetic topology

⇒ "magnetic reconnection", "tearing", etc.

Note asymmetry;  $L, \Delta$

slow  $\Delta$



①

B

but

→ Sweet-Parker is too slow

$$\text{i.e. } \frac{V}{L} \sim \frac{1}{\tau_R} \sim \frac{V_A}{L} S^{-1/2}$$

↓  
ideal

$$\left\{ \begin{array}{l} S \sim 10^8 - 10^{10} \\ \text{outer corona} \end{array} \right.$$

→ Alternatives

HW - shorter asymmetry  
i.e.  $\Delta \rightarrow L$

⇒ hyperdiffusion, electron viscosity  
electron inertia

$$\text{i.e. } \frac{\Delta}{L} \sim \frac{1}{R_{\text{eff}}}$$

$$\sim \frac{c^2}{\omega_{pe}^2} \omega_H$$

\* - anomalous resistivity →  $\frac{De}{\omega_{pe}}$   
→ low collisionality

- small scale { Shell  
                                  whistler

- waves

- shocks - Petschelt



systematic  
DKE

$$B \cdot \left[ \underline{E} + \underline{v} \times \frac{\underline{B}}{c} = n \underline{v} \right] \quad (2)$$

13.

$$\underline{0} \quad \frac{\partial F}{\partial t} + \underline{v}_z \frac{\partial F}{\partial z} - \frac{c}{B_0} \nabla \phi \times \nabla \phi \cdot \nabla F$$

$$= \frac{e \hbar}{m_0} E_{||} \frac{\partial F}{\partial v} = C(F)$$

$$\frac{e \hbar}{m_0} E_{||} = \underbrace{\langle v C(F) \rangle}_{(1)} + \underbrace{\frac{\partial J_{||}}{\partial t}}_{(2)}$$

$$\int v \cdot (-v) d(F-B)$$

$$= \underbrace{\frac{c}{B_0} \nabla \phi \times \nabla \phi \cdot \nabla J_{||}}_{(3)} = \underbrace{\frac{\partial}{\partial z} \langle v^2 F \rangle}_{(4)}$$

(1)  $\rightarrow$  resistivity  
colln.

(2)  $\rightarrow$  electron inertia  $\sim \frac{c^2}{\omega_{pe}^2}$

(3)  $\rightarrow$  electron virc.

(4)  $\rightarrow$  drift wave / pressure  
COTIA

$\checkmark \rightarrow$  waves

$$v.e. \quad \frac{\tilde{n}_e}{N} \approx \frac{e \hbar}{T} + \dots$$



# Anomalous Resistivity

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## Applications III

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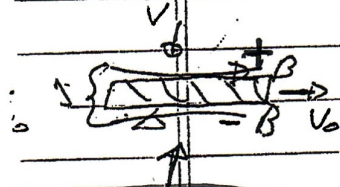
Anomalous Resistivity  $\rightarrow$  Application of QLT

Follows/expands on Galeev/Sagdeev Rev. Pl Phys 57.

$\rightarrow$  an instructive and important example of quasilinear theory is anomalous resistivity

$\rightarrow$  here try approach of kinetic current-driven ion acoustic instability (CDIA) model of anomalous resistivity via coupled micro-macro dynamics

- consider Sweet-Parker model, i.e.



$$V_L = V_{out} \Delta$$

$$V_{out} = V_A$$

$$\langle E \rangle = \langle \frac{V_B}{c} \rangle < 0$$

(into page)

$$2 \frac{V_B^2}{8\pi} L = \eta J^2 L \Delta$$

c.f. 218B notes

$$\frac{\Delta}{L} = \frac{V}{V_A}$$

$$\Rightarrow \Delta^2 = \frac{(L)(\eta)}{V_A} \Rightarrow \text{layer width}$$

$$\frac{\Delta}{L} \sim \omega_c / \Gamma_m$$

What happens as  $\eta$  decreases?

$$\frac{cB}{4\pi A} = J = n e \bar{v}$$

electron drift speed

$$\Rightarrow \bar{v} = cB / 4\pi n e L \Delta = \frac{d_{skin}^2}{\Delta} e$$

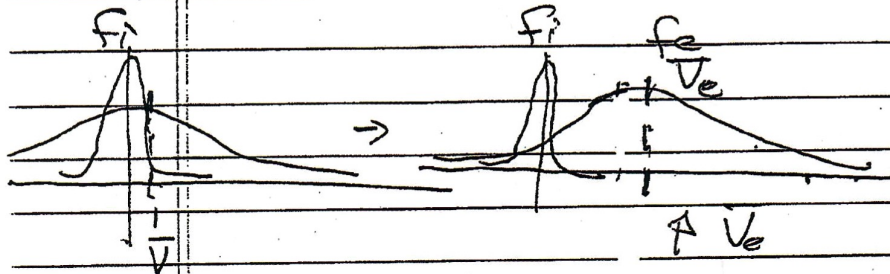


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Now  $V_e \sim B/\Delta n \Rightarrow V_e \uparrow$  as

$A \uparrow \Rightarrow$



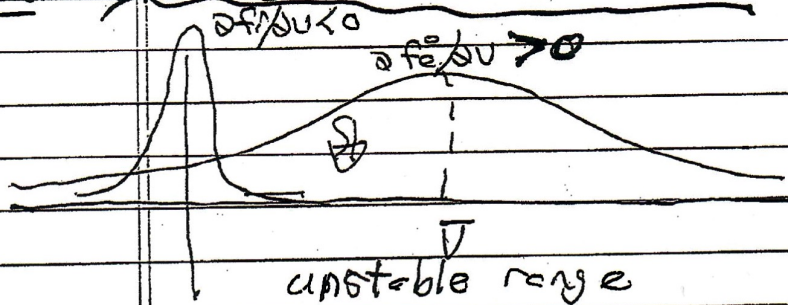
$\Delta b \rightarrow$  narrow layer  
 $n_b \rightarrow$  few charge carriers  
 $B \uparrow \Rightarrow$  stronger field (drive)

$\Rightarrow$  decreasing  $A$  raises  $V$

$\rightarrow$  up-shifts  $f_e$  centroid relative to  $f_i$

$\rightarrow$  destabilizes CDIA

i.e. classic scenario of CDIA



CDIA as  
 origin of  
 anomalous  
 resistivity

$\therefore$  expect CDIA will:

$\rightarrow$  exchange momentum between electrons and waves

so  $\rightarrow$  slow down electrons, reduce  $V_e$

$\rightarrow$  act as anomalous turbulent resistivity

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$$\text{c.e. } \left\{ \begin{aligned} \Delta^2 &= \frac{1}{V_A} (1 + \eta_A(V)) \\ V &= \frac{cB}{4\pi n e A} \end{aligned} \right. \quad \begin{aligned} &\rightarrow \text{anomalous resistivity} \\ &\text{threshold} \end{aligned}$$

How calculate?

## ② Brute Force

- confining oneself to 1D model, ignoring layer structure, however.

$$\frac{\partial F}{\partial t} + v \frac{\partial F}{\partial x} + \frac{e}{m} E \frac{\partial F}{\partial v} = -e(F) \quad \begin{aligned} &\text{here } x \rightarrow \text{vertical} \\ &v \rightarrow \text{vertical velocity} \end{aligned}$$

MeV \*  $\Rightarrow$ vertical  $\rightarrow$   
+ to layer

$$\frac{\partial \langle P_e \rangle}{\partial t} = -e \langle E \int v F \rangle = -\gamma_{e,0} n_0 m_e \bar{v}_e$$

collisional loss to core

$$\frac{\partial \langle P_e \rangle}{\partial t} = -e n_0 \langle E \rangle = -e \langle \tilde{E} \tilde{n} \rangle = -\gamma_{e,0} n_0 m_e \bar{v}_e$$

so

$$\langle E \rangle + \frac{\langle \tilde{E} \tilde{n} \rangle}{n_0} = \frac{1}{n_0} \frac{\partial \langle P_e \rangle}{\partial t} = + \frac{\gamma_{e,0} n_0 m_e \bar{v}_e}{n_0^2}$$



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at ② stationary state,

$$\langle \vec{E} \rangle + \langle \vec{E} \frac{\tilde{n}}{n_0} \rangle = \eta \langle \vec{J} \rangle$$

driving field

$$\sim \langle \vec{V} \cdot \vec{B} \rangle$$

electron  
acceleration  
by turbulence

collisional  
resistivity

$$\rightarrow \eta_L$$

anomalous resistivity

to calculate:

$$\langle \vec{E} \frac{\tilde{n}}{n_0} \rangle = \sum_{\vec{u}} + i k \vec{\phi}_{-\vec{u}} \frac{\tilde{n}_{\vec{u}}}{n_0} e$$

$$= \int d\vec{u} \sum_{\vec{u}} i k \vec{\phi}_{-\vec{u}} \vec{f}_{\vec{u}}^e$$

electron  
density perturbation

$$f_{\vec{u}}^e \rightarrow f_{\vec{u}}^{eL}$$

- quasilinear calculation

- stationarity  $\Rightarrow$  resonant transport

### ④ Conservation Argument

X

- as in ② anticipate stationarity  $\Rightarrow$   
resonant quasilinear evolution

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- recall,

$$\frac{\partial}{\partial t} \left( \epsilon^{RP} + \sum \frac{\omega_{\text{wave}}}{\omega N} \right) = 0$$

$$\frac{\partial}{\partial t} \left( p^{RP} + \sum \frac{p^{\text{wave}}}{k N} \right) = 0$$

$$\sum_n \omega_n = \omega_n \frac{\partial \epsilon}{\partial \omega} \bigg|_n \frac{|E_n|^2}{8\pi} \equiv \omega_n N_n \quad \rightarrow \# \text{ counts}$$

$$p_n^w = \frac{k}{\omega} \sum_n \omega_n = k N_n$$

as waves (CAIA) electrostatic, can ignore field momentum

so, for resonant electrons:  $k N$

$$\frac{\partial}{\partial t} p_e^{RP} = - \frac{\partial}{\partial t} p^w = - \sum_n (2\gamma_n^e) \frac{k}{\omega_n} \sum_n \omega_n$$

$\gamma_n^e \equiv$  electron (resonant) growth rate

but



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$\frac{d}{dt} P_{\text{electron}}^{\text{RAD}} \rightarrow \text{slowing down}$

$\rightarrow$  macro-representation as effective collision

ISO  $\frac{d}{dt} P_{\text{electron}}^{\text{RAD}} = -n m_e V_{\text{eff}} \bar{V}$  Frequency

effective collision frequency

slowing down by resonant scattering  
(resonant particle interaction)

$$n m_e V_{\text{eff}} \bar{V} = \sum_k (2\pi e) \frac{k}{\omega_k} \sum_n \omega_n$$

- defines  $\bar{V}$

- for macro-micro link

$$\bar{V} = \frac{cB}{4\pi n q A}$$

\* - n.b. of 2D, 3D theory, i.e. 1 dynamics  $\Rightarrow$  non-resonant scattering  $\Rightarrow$  wave driven momentum flux i.e.  $\Pi_{\perp||} \rightarrow$  1 radiation // momentum. Relation to whether interpretation of Beller II. There need include wave radiation in energy balance.

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so now have

$$\Lambda M V_{\text{eff}}(R, A) \bar{V} = \sum_k \langle \delta \delta_k^e \rangle \frac{k}{\omega_k} \Sigma_k^w \quad (1)$$

$$\Lambda^2 = \frac{6}{V_A} \left( 1 + \frac{c^3}{\omega_p^2} V_{\text{eff}} \right)$$

$\Rightarrow$  need  $\delta_k^e$ ,  $\Sigma_k^w$  and  $\langle F^e \rangle$  evolution

at simplest level, proceed via linear/quasilinear theory in 1D

- at more advanced level:

- consider 1D phase space structures

$\rightarrow$  electron/ion clumps, momentum exchange

$\rightarrow$  electron scattering off ion hole

- consider 3D  $T_{\perp}$  driven instability with electron viscosity

Now, proceed in usual fashion:

$\delta_k^e \rightarrow$  linear theory

$\Sigma_k^w \rightarrow$  nonlinear saturation

$\langle F^e \rangle \rightarrow$  Q-equation - flattening