Problem Set I: due TBA

1. Kulsrud; Chapter 3, #1 — simple, but important educational. Highlights duality of description in terms of field lines and fluid elements.

2. Kulsrud; Chapter 3, #3 — introduction to magnetic helicity

3. Kulsrud; Chapter 3, #4 — simple illustration of magnetic braking. Instructive.

4. *Electron MHD* (EMHD)

   This extended problem introduces you to EMHD and challenges you to apply what you've learned about MHD to understand the structures of a different system of fluid equations. In EMHD, the ions are stationary and the "fluid" is a fluid of electrons. EMHD is useful in problems involving fast Z-pinches, filamentation and magnetic field generation in laser plasmas, Fast Igniter, etc.

   The basic equations of EMHD are the electron momentum balance equation

   (1) $$\frac{\partial}{\partial t}\underline{\mathrm{v}}+\underline{\mathrm{v}}\cdot\underline{\nabla}\underline{\mathrm{v}}=-\frac{q}{m}\underline{E}-\frac{\nabla P}{\rho}-\frac{q}{mc}\left(\underline{\mathrm{v}}\times\underline{B}\right)-\nu\underline{\mathrm{v}},$$

   (2) $$\underline{J}=-nq\underline{\mathrm{v}},$$

   and continuity

   (3) $$\underline{\nabla}\cdot\underline{J}=0.$$

   Note that here, Ampere's law forces incompressibility of the mass flow $\rho\underline{\mathrm{v}}$. Here $\underline{\mathrm{v}}$ is the electron fluid velocity, ν is the electron-ion collision frequency, $q=|e|$, $m=m_e$. Of course, Maxwell's equations apply, but the displacement current is neglected.

i.) *Freezing-in*

Determine the freezing-in law for EMHD by taking the curl of Eqn. (1) and using the identity

$$-\underline{\mathrm{v}}\cdot\underline{\nabla}\underline{\mathrm{v}} = \underline{\mathrm{v}}\times\underline{\omega} - \underline{\nabla}\left(\mathrm{v}^2/2\right).$$

Assume the electrons have $p = p(\rho)$. Approach this problem by trying to derive an equation for "something" which has the structure of the induction equation in MHD. Discuss the physics - what is the "something" and what is it frozen into? In retrospect, why is the frozen-in quantity obvious? How is freezing-in broken?

ii.) *Large Scale Limit*

Show that for $\ell^2 >> c^2/\omega_{pe}^2$, the dynamical equations for EMHD reduce to

$$\frac{\partial B}{\partial t} + \underline{\nabla}\times\left(\frac{\underline{J}}{nq}\times\underline{B}\right) = -\nu\underline{\nabla}\times\left(\frac{\underline{J}}{nq}\right)$$

$$\underline{\nabla}\cdot\underline{J} = 0; \quad \underline{\nabla}\cdot\underline{B} = 0.$$

a) Show that density remains constant here.

b) Formulate an energy theorem for EMHD in this limit, by considering the energy content of a "blob" of EMHD fluid.

c) Discuss the frozen-in law in this limit.